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# 1 Computation

## 1.1 Arithmetic

Suppose $a, b, c$ are elements of a group $G$, and suppose $|a| = 6$ and $|b| = 7$. Express $(a^4 c^{-2} b^4)^{-1} a^3$ without using negative exponents.

## 1.2 If a group element $x$ is such that $x^{24} = e$, what are the possible orders of $x$?

## 1.3 Draw the subgroup lattice of $\mathbb{Z}_{24}$.

$$\text{Let} \quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}, \text{ and } \quad \sigma = (126)(45) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 6 & 3 & 5 & 4 & 1 & 7 \end{bmatrix} \in S_7$$

## 1.4 For each group below, if the group is finite, list all elements; if the group is infinite, describe all elements

1. $\langle$2D rotation by $240^o\rangle$ 2. $\langle A \rangle$ 3. $\langle B \rangle$ 4. $\langle AB \rangle$
5. $\langle \sigma \rangle$ 6. The symmetry group of a (non-square) rectangle, aka the "mattress group" from the day 1

## 1.5 Match the groups from Question 1.4 with the groups below

(i) $\mathbb{Z}_2$ _____ (ii) $\mathbb{Z}_3$ _____ (iii) $\mathbb{Z}_4$ _____ (iv) $\mathbb{Z}_6$ _____ (v) $\mathbb{Z}$ _____ (vi) $\mathbb{Z}_2 \times \mathbb{Z}_2$ _____

# 2 True or false

Let $G$ denote a finite group. If the statement is true, give a proof. If it is false, give a counterexample.

## 2.1 For all elements $a, b \in G$, we have $|ab| = |ba|$

## 2.2 For all elements $a, b \in G$, we have $|a| = |b^{-1}ab|$

## 2.3 For all elements $a, b \in G$, we have $|b| = |b^{-1}ab|$

# 3 Union of subgroups

## 3.1 Prove that it is impossible for a group $G$ to be the union of two proper subgroups.

(In your proof, don't assume that the group is finite)

## 3.2 Let $G$ denote the rectangle (non-square) "mattress group" (the symmetry group of a non-square rectangle). Write $G$ as the union of three proper subgroups.

# 4 Subgroup

Find all subgroups of the symmetry group of the regular triangle (the "triangle mattress" group).