Let $U(n)$ be the group of units in $\mathbb{Z}_n$ with multiplication module n as binary operation, that is,

$$\begin{aligned} U(n) &= \{x \in \mathbb{Z}_n : x \text{ has an inverse under } \cdot, \text{ multiplication modulo } n\} \\ &= \{x \in \mathbb{Z}_n : x \text{ and } n \text{ are relatively prime, meaning} \gcd(x, n) = 1\} \end{aligned}$$

1 Complete the Cayley table for $U(10)$

$\cdot$	1	3	7	9
1				
3		9		
7		1		
9				

2 Order of group and each group element

2.1 $U(10)$

Find the order of the group $U(10)$, and the order of each element in the group.

2.2 $U(12)$

Find the order of the group $U(12)$, and the order of each element in the group.

2.3 Square mattress group

Find the order of the "square mattress" group D_4, and the order of each element in the group.

2.4 Connection

Do you see a connection between the orders of the elements of a group and the order of the group?

3 Groups of order 8

Give an example of 3 different groups with 8 elements. Prove that all your three groups are different.

(Some possibilities for proving two groups are different: if G_1 is abelian and G_2 is not, then we know they are different; if G_1 has only one element of order 2 but G_2 has multiple elements of order 2, then we know they are different.)

4 Group of units of $\mathbb{Z}_n$

Let $n \geq 3$. Let $U(n)$ be the group of units in $\mathbb{Z}_n$, that is, $U(n)$. Prove that there is an element $k \in U(n)$ of order 2, that is, $k^2 = 1$ and $k \neq 1$.

5 Subgroup

Find all subgroups of the symmetry group of the regular triangle (the "triangle mattress" group).

6 Subgroup

Let H be a subgroup of G and let $g \in G$. Define gHg^{-1} to be the set $gHg^{-1} = \{ghg^{-1} : h \in H\}$. Prove that gHg^{-1} is a subgroup of G.