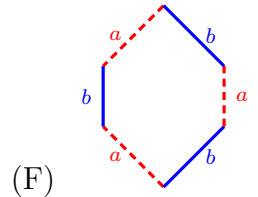
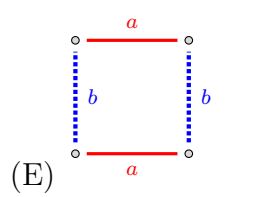
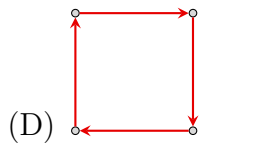
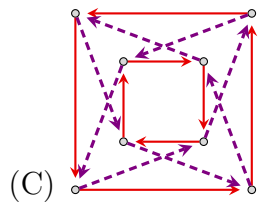
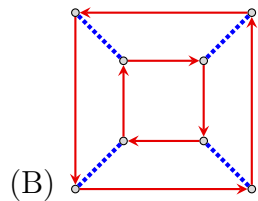
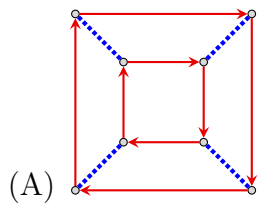
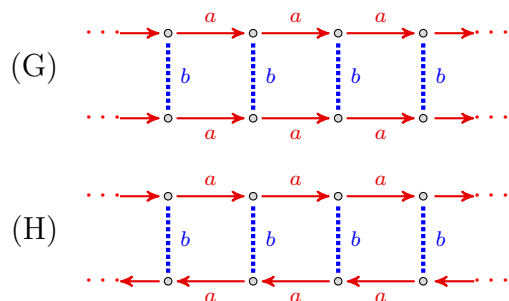


1. (a) Consider the subgroup  $H = \{(1), (1, 2)\}$  of  $S_3$ . Are all left cosets of  $H$  the same as all right cosets of  $H$ ?  
(Possible answers: yes/no)
  - (b) Consider the subgroup  $J = \{(1), (123), (132)\}$  of  $S_3$ . Are all left cosets of  $J$  the same as all right cosets of  $J$ ?  
(Possible answers: yes/no)
  - (c) Let  $n \geq 3$ . Consider the alternating subgroup  $A_n$  in the symmetric group  $S_n$ . Are all left cosets of  $A_n$  in  $S_n$  the same as the right cosets?  
(Possible answers: yes/no)
2. (a) What do the arrows in a Cayley diagram represent?  
(b) What do the vertices in a Cayley diagram represent?
3. Below are Cayley diagrams of eight different groups (none of them has the same group structure). For each Cayley diagram, determine whether the corresponding group is abelian or not abelian.  
(Possible answers: abelian/not abelian)





4. For each of the following group and its generating set, draw the Cayley graph on your own paper. Which Cayley graph from Question 3 is it?

(Possible answers: A, B, C, D, E, F, G, H)

- (i) The group  $S_3$  with minimal generating set the transpositions  $(1\ 2)$  and  $(2\ 3)$ .
- (ii) The subgroup of  $S_4$  generated by the transpositions  $(1\ 2)$  and  $(3\ 4)$ .
- (iii) The subgroup of  $S_6$  generated by the 4-cycle  $(1\ 5\ 3\ 6)$ .
- (iv) The direct product  $\mathbb{Z}_4 \times \mathbb{Z}_2$  with two generators  $(1, 0)$  and  $(0, 1)$ .
- (v) The direct product  $\mathbb{Z} \times \mathbb{Z}_2$  with generating set  $\{(1, 0), (0, 1)\}$ .
- (vi) The dihedral group  $D_4$  generated by  $R$  (the counterclockwise rotation by  $90^\circ$ ) and  $f$  (a flip along the vertical mirror).