Math 118 Calculus Ia **Test 4**

This is a closed-book, closed-notes, no-calculators test. There are 60 points possible.
Fractions and roots in answers are fine; so are negative and fractional exponents.
Use scratch paper as needed, but any work that you want graded should be written legibly on this test paper.

(1 pt) Sign below to indicate your pledge. If your signature is difficult to read, please print your name as well.

I pledge that I will not give, accept, or tolerate others' use of unauthorized aid in completing this work.

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(9 pt) 1. Use derivative rules to find f'(x). review sec 4.4 - The Chain Rule. / General Power Rule
Don't spend a lot of time simplifying once all the differentiation steps are completed.

a. $f(x) = (x^2 - x - 1)^6$ $f'(x) = 6(x^2 - x - 1)^5 \cdot (2x - 1)$

b. $f(x) = \sqrt{x^4 + 16}$ $f'(x) = \frac{1}{2}(x^4 + 16)^{-1/2} \cdot 4x^3$

c. $f(x) = x^2(3 - x)^4$ $f'(x) = x^2 \cdot \frac{d}{dx}(3-x)^4 + (3-x)^4 \cdot \frac{d}{dx}(x^2)$ product rule

$= x^2 \cdot 4(3-x)^3 \cdot (-1) + (3-x)^4 \cdot 2x$ ← fine

or $-4x^2(3-x)^3 + 2x(3-x)^4$ ← also fine.

(2 pt) 2. If f is a differentiable function and $g(t) = f(1/t)$ then $g'(t) = \cdots$ 4.4 Chain Rule

a. $f'(1/t)$ b. $\frac{-1}{t^2} \cdot f'(1/t)$ c. $f'(-1/t^2)$ d. $\frac{1}{f'(t)}$ e. $f(1/t) \cdot f'(1/t)$

(2 pt) 3. Suppose y is a differentiable function of x. 4.4 Chain Rule / 4.5 Implicit Diff.

Choose the correct expression for $\dfrac{d}{dx}\left[2y^7\right]$ from the following:

a. $14y^6\dfrac{dy}{dx}$ b. $14y^6$ c. 0 d. $2y^7\dfrac{dy}{dx}$ e. $7y^6$

use the product rule here.

(8 pt) 4. Use implicit differentiation to find $\dfrac{dy}{dx}$ for the curve $x^2 + 3xy + y^3 = 15$.

diff. with respect to x:

$$2x + \underbrace{3x \cdot \frac{dy}{dx} + 3y}_{\text{from product rule}} + 3y^2\frac{dy}{dx} = 0$$

isolate...

$$\frac{dy}{dx} = \frac{-2x-3y}{3x+3y^2}$$

sign of $f'(x)$: ⊖ 0 ⊕ 0 ⊕

$\longleftarrow$ -1 —— 2 $\longrightarrow$

(4 pt) 5. True/False I.

You are given that $f(x)$ is a function whose derivative is $f'(x) = (x-2)^2(x+1)$.

$x=-1$ and $x=2$ are critical numbers of f.

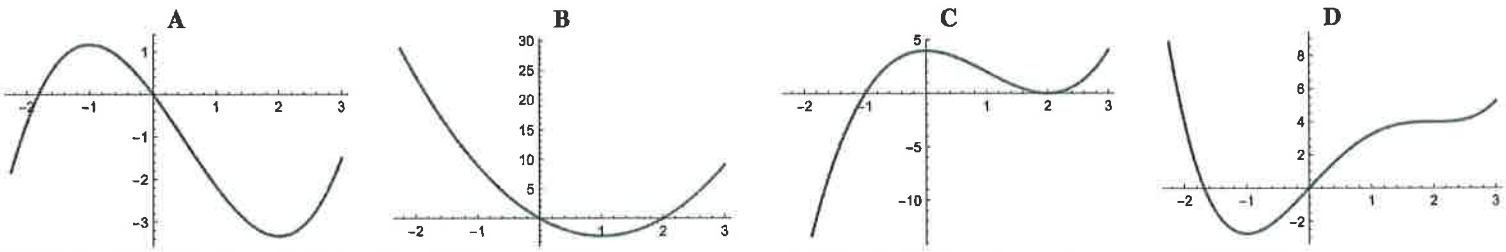
T a. f is increasing on $(-1, 2)$. because f' is positive on $(-1,2)$.

F b. $f'(x)$ changes sign at $x = 2$. $f'(x)$ is positive on both sides of $x=2$

T c. f has a relative minimum at $x = -1$. f changes from decreasing to increasing at $x=-1$, like this:

$x=-1$

See section 5.3 esp. Thm 5.5 and Thm 5.6



(6 pt) 6. As in the previous problem, $f(x)$ is a function whose derivative is $f'(x) = (x-2)^2(x+1)$.
Choose from the graphs above to answer the following:

D i. Which of the above is the graph of $f(x)$? decreasing on $(-\infty,-1)$, inc. on $(-1,2)$ & $(2,\infty)$.

C ii. Which of the above is the graph of $f'(x)$? zeros at $x=-1$ and $x=2$, positive on both sides of $x=2$.

B iii. Which of the above is the graph of $f''(x)$? it's positive where f' is increasing and negative on $(0,2)$ where f' is decreasing.

Sec 5.3, like #5.

(6 pt) 7. A moving object has its position at time t given by $f(t) = (1/3)t^3 - 5t^2 + 16t$.
Find the time interval on which the object is moving backward (in other words, the time interval during which the position function is decreasing).

Sec 5.3.

$$f'(t) = t^2 - 10t + 16$$

critical numbers of f:
$$t^2 - 10t + 16 = 0$$
$$(t-8)(t-2) = 0$$
$$t = 2 \text{ or } t = 8.$$

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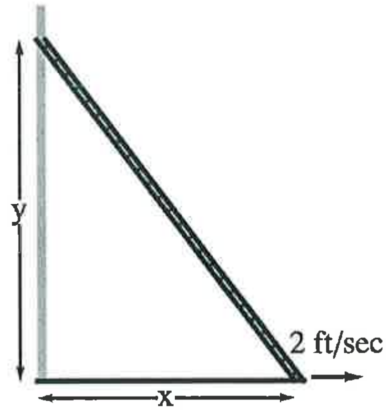
$f'(t)$ - that is, velocity - is negative on $(2,8)$.

So the object is moving backward from time $t=2$ to time $t=8$.

(10 pt) 8. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Refer to the figure for the notation used in the following questions:



Sec 4.6
exercise #27, in fact.

Also see the video from that day on Moodle.

i. Which of the following represents the "2 feet per second" given in the description above?

a. x b. dx/dt c. y d. dy/dt e. dy/dx

"the rate at which x is changing"

ii. At the instant shown in the figure, dy/dt is...

a. positive   b. negative   c. zero   d. undefined

As the ladder slides, y is decreasing, so dy/dt should be negative.

iii. How fast is the top of the ladder moving down the wall when the base is 15 feet from the wall? You may need the useful arithmetic fact that $15^2 + 20^2 = 25^2$.

x and y are related by the Pythagorean Theorem.

$$x^2 + y^2 = 25^2$$

diff. (with respect to time) to relate the rates:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0.$$

Isolate the quantity you're trying to find.

$$\frac{dy}{dt} = \frac{-x}{y}\cdot\frac{dx}{dt}$$

At the time of interest,

$x = 15$ (given)

$y = 20$ (from Pyth. Thm & the given fact).

and $\frac{dx}{dt} = 2$ (given).

so $$\frac{dy}{dt} = \frac{-15}{20}\cdot 2 = \frac{-15}{10} = \frac{-3}{2} \text{ ft/sec.}$$

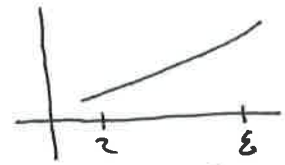
Sec. 5.1 - Extreme Value Thm (Thm 5.1).

(4 pt) 9. True/False II. Assume that f is continuous on the interval $[2, 8]$.

T a. $f(x)$ must have an absolute maximum value on $[2, 8]$ EVT

F b. $f(x)$ must have at least one critical number in $[2, 8]$.

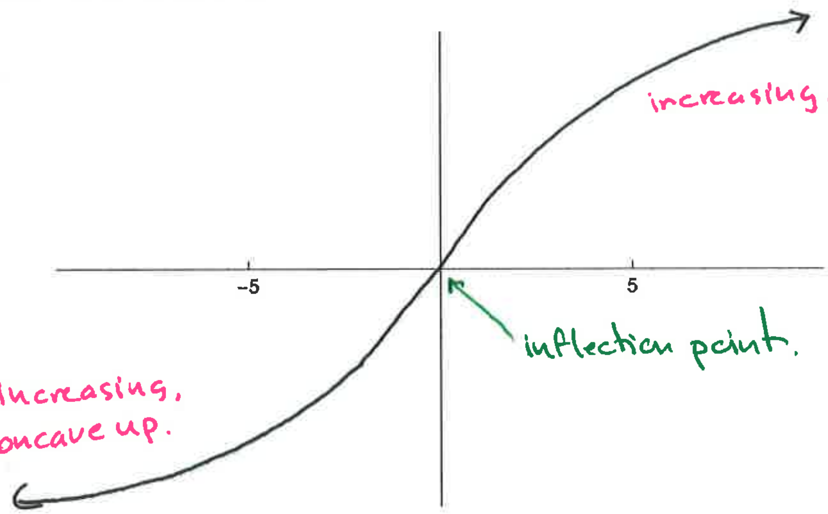
T c. $f(x)$ must have an absolute minimum value on $[2, 8]$. EVT



(b) - For ex., f might be incr. over the whole interval and have no critical numbers.

(3 pt) 10. Use the provided axes to sketch the graph of a function which is increasing everywhere, concave up on $(-\infty, 0)$ and concave down on $(0, +\infty)$.

For ex:



See sec. 5.4.

(6 pt) 11. The function $f(x) = x^3 - 5x^2$ has exactly one inflection point. Find its x-coordinate.

Sec 5.4.
Thm 5.7/5.8

$f'(x) = 3x^2 - 10x$

$f''(x) = 6x - 10$

the inflection point must be where

$6x - 10 = 0$

so, $x = 5/3$.