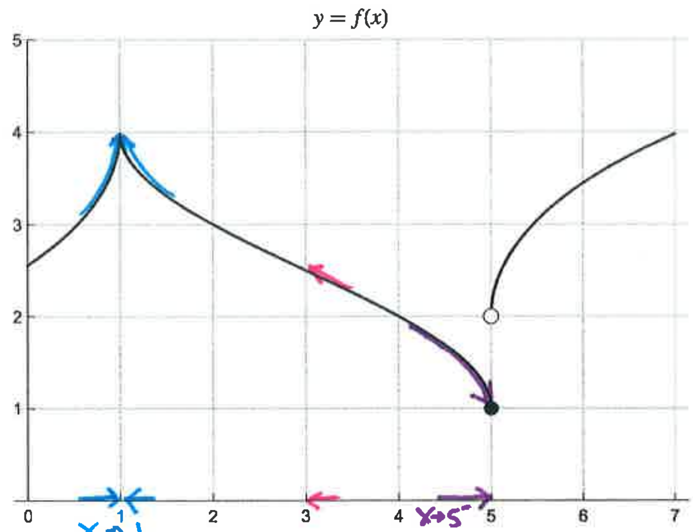
118 Cumulative
Test Sol'ns



(3 pt) 1. Read the values of the following limits from the graph. Answer with $+\infty$ or $-\infty$ if appropriate.

a. $\lim_{x\to 5^-} f(x) = 1$ "x approaches 5 from the left"

b. $\lim_{x\to 1} f(x) = 4$

c. $\lim_{x\to 3^+} \frac{f(x)}{3-x}$ $= -\infty$

about 2.5

$\to 0^-$

(6 pt) 2. Use the graph of $f(x)$ shown above to answer the following:

a. What is the net change in f over the interval $[1, 5]$? $f(5) - f(1) = 1 - 4 = -3$

note that the sign (negative) is important!

b. What is the average rate of change in f over the interval $[1, 5]$? $\frac{1-4}{5-1} = \frac{-3}{4}$

c. Which of the following is closest to the value of $f'(3)$? (Circle one value)

-3.1 -0.5 0.0 0.7 0.2

slope of the tangent line at $x=3$. draw it in, or visualize it, and estimate the slope.

(6 pt) 3. Evaluate the following limits. Answer with $+\infty$ or $-\infty$ if it is appropriate.

a. $\lim_{t\to 0} \frac{t^2-4t}{t^2+8t}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$ $= \lim_{t\to 0} \frac{\cancel{t}(t-4)}{\cancel{t}(t+8)} = \frac{0-4}{0+8} = \frac{-4}{8}$ or $\frac{-1}{2}$

Factor and cancel.

b. $\lim_{x\to 4} \frac{x^2-16}{x+4}$ $\begin{matrix}\to 0\\ \to 4\end{matrix}$ $= \frac{0}{4} = 0.$ just by direct substitution.

(3 pt) 4. Evaluate **ONE** of the following limits.
Cross out the one that you are not evaluating.

a. $\lim_{x\to 1} \frac{6x^3+x^2+5x-12}{x-1}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$ or b. $\lim_{x\to 18} \frac{\sqrt{2x}-6}{x-18}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(6x^2+7x+12)}{\cancel{(x-1)}}$$

$$= 6+7+12 = 25.$$

Factoring:

$$\begin{array}{r} 6x^2+7x+12 \\ x-1 \enclose{longdiv}{6x^3+x^2+5x-12} \\ -(6x^3-6x^2) \\ \hline 7x^2+5x-12 \\ -(7x^2-7x) \\ \hline 12x-12 \\ -(12x-12) \\ \hline 0. \end{array}$$

$$= \lim_{x\to 18} \frac{\sqrt{2x}-6}{x-18} \cdot \frac{\sqrt{2x}+6}{\sqrt{2x}+6}$$

$$= \lim_{x\to 18} \frac{2x-36}{(x-18)(\sqrt{2x}+6)}$$

$$= \lim_{x\to 18} \frac{2\cancel{(x-18)}}{\cancel{(x-18)}(\sqrt{2x}+6)}$$

$$= \frac{2}{\sqrt{36}+6} = \frac{2}{12} \text{ or } \frac{1}{6}$$

(3 pt) 5. Give an example of a rational function with vertical asymptotes at both $x = 1$ and $x = 3$.

For ex. $\dfrac{1}{(x-1)(x-3)}$ - anything that has $x=1$ and $x=3$ as zeros of the denom. but not the numerator will do.

or $\dfrac{x^8+13}{(x-1)^5(x-3)^3}$

(6 pt) 6. True/False. Partial credit: 5 correct = 4 points; 4 correct = 2 points.

You are given that $f(x)$ is a polynomial of degree 4, $f(0) = -10$, and $f(80) = 30$.

F a. $f(x)$ must have the same average rate of change over every interval $[a, b]$.

T b. There must be at least one value c in $(0, 80)$ where $f'(c) > 0$.

F c. There must be at least one value c in $(0, 80)$ where $f'(c) = 0$.

T d. $f(x)$ must have at least one real zero.

T e. $f'(x)$ must have at least one real zero.

T f. f must be differentiable everywhere.

a. That would be true if and only if f is a linear function.

b. MVT: Actually, there must be some c in $(0, 80)$ where $f'(c) = \dfrac{30-(-10)}{80-0} = \dfrac{40}{80}$, which is positive.

more conceptually, the function increased over the interval $[0, 80]$, so there must have been some point at which the rate of change was positive!

c. why should there be? f' could, for example, be positive at every point in the interval. Rolle's Theorem doesn't apply here since $f(0) \neq f(80)$.

d. True by IVT because f is continuous, $f(0) < 0$ and $f(80) > 0$.

e. True because $f'(x)$ is degree 3, and odd degree polynomials always have at least one real zero.

f. All polynomials are diff'ble and continuous everywhere.

(8 pt) 7. Find an equation for the tangent line to $y = x^3 - x$ at $x = 2$.

deriv. $\frac{dy}{dx} = 3x^2 - 1$ so slope at $x=2$ $= 3 \cdot 2^2 - 1 = \underline{11}$.

and $y = 2^3 - 2 = 6$, so the point is $(2,6)$.

tan. line: $y - 6 = 11(x-2)$.

(8 pt) 8. The position function for a departing train as it leaves the station is given by

$$f(t) = \frac{5}{2}t^2 + 4t$$

meters, at time t seconds. Determine the time at which the train reaches a velocity of 54 m/s, and determine the train's position at that time.

velocity Function: $f'(t) = 5t + 4$

solve: $5t + 4 = 54$

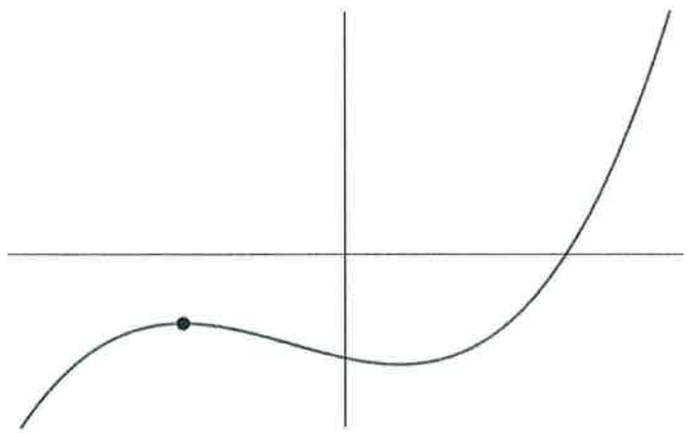
$\Rightarrow$ $t = 10$ sec.

the pos'n at time $t=10$ is

$f(10) = \frac{5}{2} \cdot 100 + 4 \cdot 10 = 250 + 40 = 290$ m

(From the station)

(8 pt) 9. The graph of $f(x) = x^3 + x^2 - x - 3$ is shown here:



a. Compute $f''(x)$.

First, $f'(x) = 3x^2 + 2x - 1$

then $f''(x) = 6x + 2$

b. This function has two turning points. Find the exact x- and y-coordinates of the leftmost turning point (the one which is marked on the graph).

they occur at points where the tangent line is horizontal,

so solve:

$$3x^2 + 2x - 1 = 0$$

$$(3x - 1)(x + 1) = 0$$

$\cancel{x = \frac{1}{3}}$ or $\boxed{x = -1}$

that's the rightmost one.

At $x =$ $y = (-1)^3 + (-1)^2 - (-1) - 3 = -1 + 1 + 1 - 3 = -2$.

so the point is $(-1, -2)$.

(3 pt) 10. Compute $f'(2)$ if $f(x) = \frac{1}{x^3}$

think $f(x) = x^{-3}$, so $f'(x) = -3x^{-4} = \frac{-3}{x^4}$

And so $f'(2) = \frac{-3}{2^4}$, or $\frac{-3}{16}$.

(2 pt) 11. Suppose f is a differentiable function, and $g(x) = x^2 f(x)$.
Express $g'(x)$ in terms of f and/or f' (just circle the letter of your choice):

a. $x^2 f'(x) + 2xf(x)$ b. $f'(x^2)f'(x)$ c. $2xf'(x)$ d. $2xf'(1)$ e. $2x + f'(x)$

Product rule: $x^2 \cdot f'(x) + f(x) \cdot 2x$,

more normally written $x^2 f'(x) + 2x f(x)$.

(2 pt) 12. Suppose f is a differentiable function. Then $\frac{d}{dx}\left[f\left(x^2\right)\right] = \cdots$

a. $f'(x^2)$ b. $2xf'(x^2)$ c. $2xf'(x)$ d. $f(2x)$ e. $f'(2x)$

chain rule: $\frac{d}{dx}\left[f(\sim)\right] = f'(\sim) \cdot \frac{d}{dx}\left[\sim\right]$ here, fill in x^2 for the $\sim$

(2 pt) 13. Suppose y is a function of x (as we would do in implicit differentiation).

Choose the correct expression for $\frac{d}{dx}\left[x^3y^3\right]$ from the following:

a. $3x^2y^3$

b. $3x^2y^2\frac{dy}{dx}$

c. $x^3(3y^2) + y^3(3x^2)$

d. $x^3(3y^2)\frac{dy}{dx} + y^3(3x^2)$

e. $3x^2 + 3y^2\frac{dy}{dx}$

overall, it is the product rule:

$x^3 \cdot \frac{d}{dx}(y^3) + y^3 \cdot \frac{d}{dx}(x^3)$

$= x^3 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 3x^2$

"Ich mag die Leibnizregel!"