

# Math 118 Calculus Ia: Skills 4 - Key

#1.

$$a) \frac{x^3 - x - 6}{(x-2)} = \frac{(x^2 + 2x + 3)(x-2)}{(x-2)} = \boxed{x^2 + 2x + 3}$$

$$b) \frac{x^4 + 2x^2 - 3}{(x+1)(x+2)} = \frac{(x+1)(x^3 - x^2 + 3x + 3)}{(x+1)(x+2)} = \boxed{\frac{x^3 - x^2 + 3x + 3}{x+2}}$$

$$c) \frac{x^3 - 10x + 24}{x^2 - 16} = \frac{(x^2 - 4x + 6)(x+4)}{(x-4)(x+4)} = \boxed{\frac{x^2 - 4x + 6}{x-4}}$$

#2.

$$a) \frac{\frac{1}{x+4} - \frac{1}{x}}{4} = \frac{\frac{x - (x+4)}{(x+4)(x)}}{4} = \frac{\overbrace{-4}^{\text{fine}}}{4(x+4)(x)} = \boxed{\frac{-1}{(x+4)(x)}} \quad \text{even finer!}$$

$$b) \frac{\frac{1}{x^2} - \frac{1}{y^2}}{x-y} = \frac{\frac{y^2 - x^2}{x^2 y^2}}{x-y} = \frac{y^2 - x^2}{(x^2 y^2)(x-y)} = \boxed{\frac{-(x+y)}{x^2 y^2}}$$

$$c) \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - x - \Delta x}{(x)(x+\Delta x)}}{\Delta x} = \frac{x - x - \Delta x}{x \cdot \Delta x \cdot (x+\Delta x)} = \frac{-\Delta x}{x \cdot \Delta x \cdot (x+\Delta x)} = \boxed{\frac{-1}{x(x+\Delta x)}}$$

Fill in the details here!

#3.

$$a) (3 - \sqrt{x})(3 + \sqrt{x}) = (3)(3) + 3\sqrt{x} - 3\sqrt{x} - (\sqrt{x})(\sqrt{x}) = \boxed{9 - x}$$

$$b) (\sqrt{2} + \sqrt{z})(\sqrt{2} - \sqrt{z}) = (\sqrt{2})(\sqrt{2}) + (\sqrt{2})(\sqrt{z}) - (\sqrt{z})(\sqrt{2}) - (\sqrt{z})(\sqrt{z}) = \boxed{2 - z}$$

$$c) (\sqrt{x-\Delta x} - \sqrt{x})(\sqrt{x-\Delta x} + \sqrt{x}) = (\sqrt{x-\Delta x})(\sqrt{x-\Delta x}) + (\sqrt{x-\Delta x})(\sqrt{x}) - (\sqrt{x})(\sqrt{x-\Delta x}) - (\sqrt{x})(\sqrt{x}) \\ = x - \Delta x - x = \boxed{-\Delta x}$$

#4.

$$a) \lim_{x \rightarrow 1} x^2 - 2(x+1) = (1)^2 - 2(1+1) = 1 - 4 = \boxed{-3}$$

↳ direct substitution.

$$b) \lim_{x \rightarrow 0} \frac{x+6}{(x-1)(x-2)} \stackrel{\text{f.o.s.}}{=} \frac{0+6}{(0-1)(0-2)} = \frac{6}{(-1)(-2)} = \frac{6}{2} = \boxed{3}$$

$$c) \lim_{x \rightarrow 1} \frac{x^2 - 1}{(x-1)} \stackrel{\text{f.o.s.}}{=} \frac{(x+1)(x-1)}{x-1} \stackrel{\text{f.o.s.}}{=} \lim_{x \rightarrow 1} x+1 = 1+1 = \boxed{2}$$

↳ lim x→1