

#1.

$$a) (x + \Delta x)^2 = x^2 + x(\Delta x) + x(\Delta x) + (\Delta x)^2 = \boxed{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$b) (x + \Delta x)^3 = (x + \Delta x)(x^2 + 2x(\Delta x) + (\Delta x)^2) = x^3 + 2x^2(\Delta x) + x(\Delta x)^2 + x^2(\Delta x) + 2x(\Delta x)^2 + (\Delta x)^3$$

$$= \boxed{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3}$$

$$c) (x + \Delta x)^4 = (x + \Delta x)(x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3)$$

$$= x^4 + 3x^3(\Delta x) + 3x^2(\Delta x)^2 + x(\Delta x)^3 + x^3(\Delta x) + 3x^2(\Delta x)^2 + 3x(\Delta x)^3 + (\Delta x)^4$$

$$= \boxed{x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4}$$

$$d) (x + \Delta x)^5 = (x + \Delta x)(x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4)$$

$$= x^5 + 4x^4(\Delta x) + 6x^3(\Delta x)^2 + 4x^2(\Delta x)^3 + x(\Delta x)^4 + x^4(\Delta x) + 4x^3(\Delta x)^2 + 6x^2(\Delta x)^3 + 4x(\Delta x)^4 + (\Delta x)^5$$

$$= \boxed{x^5 + 5x^4(\Delta x) + 10x^3(\Delta x)^2 + 10x^2(\Delta x)^3 + 5x(\Delta x)^4 + (\Delta x)^5}$$

#2.

$$a) \frac{4(\Delta t)^3 + 3(\Delta t)^2 + 2(\Delta t)}{5(\Delta t)} = \frac{(\Delta t)(4(\Delta t)^2 + 3(\Delta t) + 2)}{5(\Delta t)} = \boxed{\frac{4(\Delta t)^2 + 3(\Delta t) + 2}{5}}$$

$$b) \frac{2(\Delta x)^3 + 6x^2(\Delta x) - 2(\Delta x)}{2(\Delta x)} = \frac{2(\Delta x)((\Delta x)^2 + 3x^2 - 1)}{2(\Delta x)} = \boxed{(\Delta x)^2 + 3x^2 - 1}$$

$$c) \frac{x^4 - 3x^2 - 54}{(x+3)(x+1)} = \frac{(x^2+6)(x-3)(x+3)}{(x+3)(x+1)} = \frac{(x^2+6)(x-3)}{(x+1)} = \frac{x^3 - 3x^2 + 6x - 18}{x+1}$$

$$\#3. a) \frac{\sqrt{x+1} - 1}{x} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} = \frac{(x+1) + \sqrt{x+1} - \sqrt{x+1} - 1}{x(\sqrt{x+1} + 1)} = \frac{x+1-1}{x(\sqrt{x+1} + 1)} = \boxed{\frac{1}{\sqrt{x+1} + 1}}$$

$$b) \frac{\sqrt{b+2(\Delta x)} - \sqrt{b}}{\Delta x} \cdot \frac{\sqrt{b+2(\Delta x)} + \sqrt{b}}{\sqrt{b+2(\Delta x)} + \sqrt{b}} = \frac{(b+2(\Delta x)) + \sqrt{b}(\sqrt{b+2(\Delta x)}) - \sqrt{b}(\sqrt{b+2(\Delta x)}) - b}{(\Delta x)(\sqrt{b+2(\Delta x)} + \sqrt{b})}$$

$$= \frac{b + 2(\Delta x) - b}{(\Delta x)(\sqrt{b+2(\Delta x)} + \sqrt{b})} = \boxed{\frac{2}{\sqrt{b+2(\Delta x)} + \sqrt{b}}}$$

$$c) \frac{\sqrt{8+b} - \sqrt{8+a}}{(b-a)} \cdot \frac{\sqrt{8+b} + \sqrt{8+a}}{\sqrt{8+b} + \sqrt{8+a}} = \frac{(8+b) - (\sqrt{8+b})(\sqrt{8+a}) - (\sqrt{8+b})(\sqrt{8+a}) + (8+a)}{(b-a)(\sqrt{8+b} + \sqrt{8+a})}$$

$$= \frac{8+b-8-a}{(b-a)(\sqrt{8+b} + \sqrt{8+a})} = \boxed{\frac{1}{\sqrt{8+b} + \sqrt{8+a}}}$$