Math 118 Calculus Ia **Quiz 1**

This is an open-book, open-notes quiz, and you may take as much time as you like. However, work alone; tutors, other students, internet, and so on are off limits. Write your answers on this quiz sheet and have it ready to turn in at the beginning of class of Thursday September 22.

(12 pt) 1. Let f be the function defined by $f(x) = x^3 - 3x$

a. Compute the average rate of change in f over the same interval, $[-1, 1]$.

$$\frac{f(1) - f(-1)}{1-(-1)} = \frac{-2-2}{1+1} = \frac{-4}{2} = -2$$

b. Write an equation for the secant line which meets the graph of f at $x = 1$ and $x = 2$.

points: $(1, f(1)) = (1, -2)$ and $(2, f(2)) = (2, 2)$ slope: $\frac{2-(-2)}{2-1} = \frac{4}{1} = 4$

line: $y - 2 = 4(x-2)$ or $y = 4x - 6$

c. Evaluate $f(x + \Delta x)$

(Your answer should be a simple formula involving the two variables x and Δx.)

think: $f(\square) = (\square)^3 - 3(\square)$.

so: $f(x+\Delta x) = (x+\Delta x)^3 - 3(x+\Delta x)$.

d. Evaluate and simplify the following quotient as much as possible:

(To fully simplify it, you'll probably need to expand a power in the numerator, simplify by combining like terms, and factor. Show your steps.)

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - 3(x+\Delta x) - (x^3 - 3x)}{\Delta x}$$

expand and combine like terms (Fill in more details!)

$$= \frac{\Delta x^3 - 3\Delta x + 3\Delta x \cdot x^2 + 3 \cdot \Delta x^2 \cdot x}{\Delta x}$$

(Factor out) and cancel the common factor.

$$= \frac{\cancel{\Delta x}\left[\Delta x^2 - 3 + 3x^2 + 3 \cdot \Delta x \cdot x\right]}{\cancel{\Delta x}}$$

$$= (\Delta x)^2 - 3 + 3x^2 + 3 \cdot \Delta x \cdot x$$

(4 pt) 2. True/False.

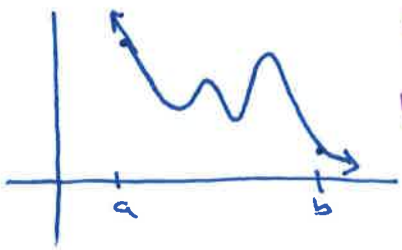
Scoring: 3 correct = 4pt, 2 correct = 2pt; otherwise, 0pt.

F a. If the average rate of change in f over $[0, 10]$ is negative, then f must be decreasing over $[0, 10]$.

T b. If f is a linear function, then f has the same average rate of change over every possible interval.

F c. Every polynomial function has at least one real zero.

For (a), consider a function like this:



av. rate of change over $[a,b]$ is negative but f is not decreasing over $[a,b]$.

For (c), consider an example like x^2+1.

(2 pt) 3. Which of the following functions has / have a constant rate of change (the same average rate of change over every possible interval)?

Circle *all* of the functions that fit the description.

~~a. x^2~~ b. $\cancel{\sqrt{x}}$ c. $(\cancel{x-2})(\cancel{x}+3)$ ~~d. $5/x$~~ e. $3x + 13$

only the linear function will have the same rate of change over every interval.

(2 pt) 4. Give an example of a polynomial of degree 3 which rises to the right, falls to the left, and has $x = 1$ as a zero.

for example, $f(x) = x^3 - 1$

many examples are possible.

you could write your polynomial in factored form, too.

for ex., $(x-1)^3$ and $(x-1)(x^2+10)$ are also good answers.