118 Problems 1 - sol'us!

$f(x) = x^2 + x + 1.$

1. The net change in f over $[1, b]$ is
$f(b) - f(1) = (b^2+b+1) - (1^2+1+1) = \underline{b^2+b-2}$

so, solve:

$$b^2+b-2 = 10$$
$$b^2+b-12 = 0$$
$$(b+4)(b-3) = 0$$

so $\cancel{b=-4}$ or $b = 3.$

? doesn't make sense in the problem.

indeed. (to check),
$f(3) - f(1) = 13 - 3 = +10$ units.

b. Let's choose a different left endpoint and see if we can do the same thing.
net change in f over $[0, b]$ is

$f(b) - f(0) = (b^2+b+1) - 1 = b^2+b$

solve to find the value of b that makes this 10:

$$b^2+b = 10$$
$$b^2+b-10 = 0$$

QF: $b = \cancel{\frac{-1-\sqrt{41}}{2}}$ doesn't make sense. or $b = \frac{-1+\sqrt{41}}{2}$

so f has a net change of +10 over $\left[0, \frac{-1+\sqrt{41}}{2}\right]$, too.

(there are many other possibilities.
choose any left endpoint you want,
& you can solve for the corresponding right endpoint.)

For ex., $[4, 5]$ is another possibility with "nice" numbers.

(remember, $f(x) = x^2 + x + 1$.)

c. The average rate of change in f over $[0, b]$ must be 10, so solve:

$$\frac{f(b) - f(0)}{b - 0} = 10$$

$$\frac{b^2 + b}{b} = 10 \quad \text{so} \quad b^2 + b = 10b.$$

$$b^2 - 9b = 0$$
$$b(b - 9) = 0$$
$$\cancel{b = 0} \quad \text{or} \quad b - 9 = 0, \quad \text{so} \quad \underline{b = 9}.$$

doesn't make sense in the problem.

The average rate of change in f over $[0, 9]$ is 10.

d. Find another interval with the same av. rate. Let's try $x = 1$ as the left endpoint.

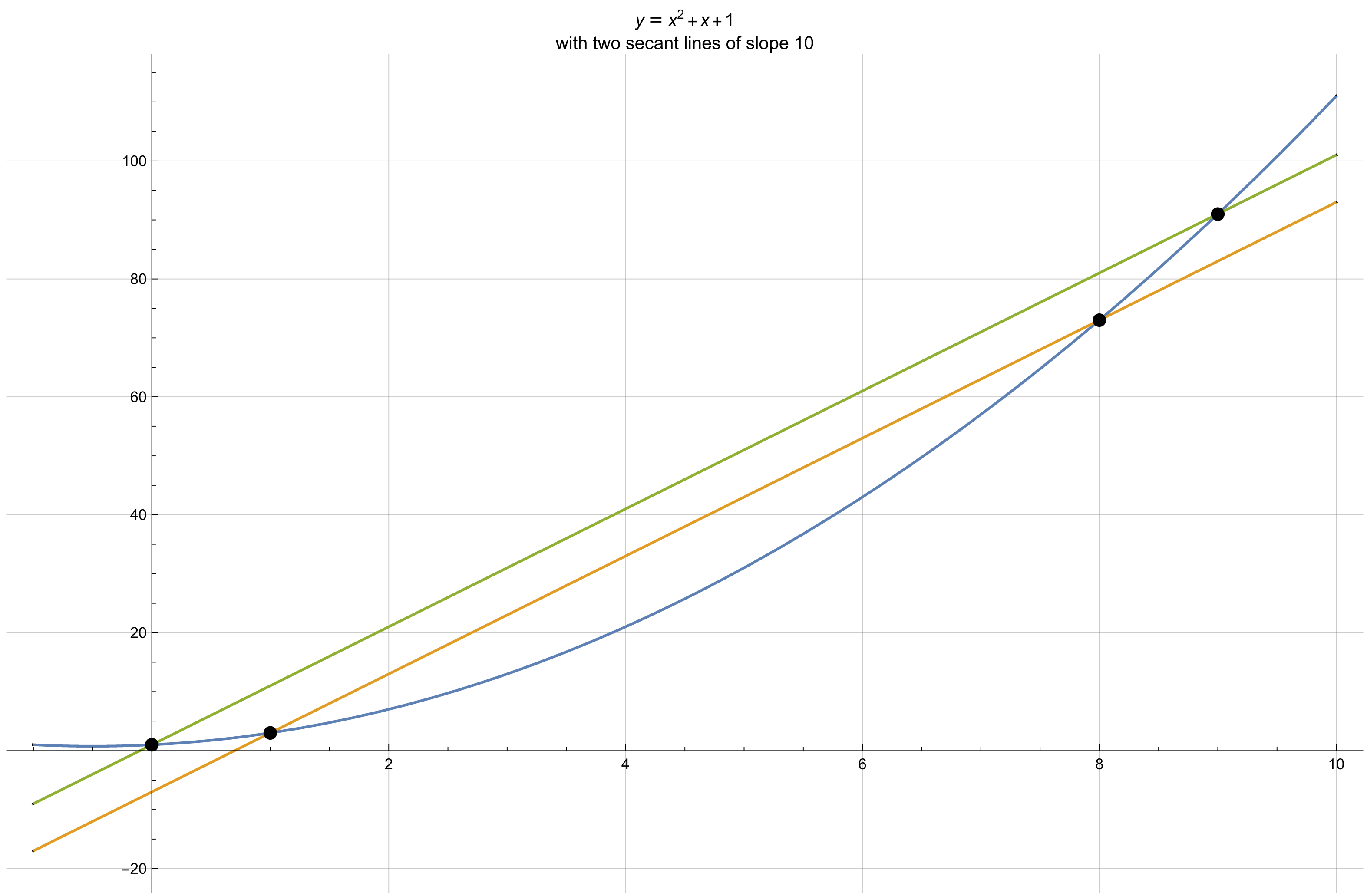
solve:
$$\frac{f(b) - f(1)}{b - 1} = 10$$

$$\frac{b^2 + b - 2}{b - 1} = 10$$

$$b^2 + b - 2 = 10b - 10$$

$$b^2 - 9b + 8 = 0$$
$$(b - 8)(b - 1) = 0$$
$$\cancel{b = 1} \quad \text{or} \quad \underline{b = 8}.$$

The av. rate of change over $[1, 8]$ is also 10.


$y = x^2 + x + 1$
with two secant lines of slope 10
100
80
60
40
20
−20
2
4
6
8
10