1. Let $f(x) = x^3 - 3x^2 + 4$. We'll analyze the increasing/decreasing behavior of f, and its relative extrema.

a. Compute $f'(x)$ and solve to find any critical numbers.
(You should be able to factor $f'(x)$ pretty easily.)

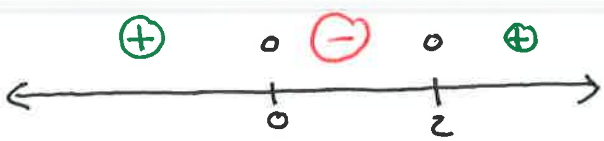
$F'(x) = 3x^2 - 6x$

critical numbers where $3x^2 - 6x = 0$

$3x(x-2) = 0$ ⊛

either $x = 0$ or $x = 2$

b. Determine the sign of $f'(x)$ on the intervals cut out by the critical numbers.



sign of $F'(x)$

-it's easiest to determine the signs using the Factored Form of $F(x)$ that appears at ⊛

c. Give the correct intervals to complete the following:

f is increasing on... $(-\infty, 0)$ and $(2, \infty)$

and f is decreasing on... $(0, 2)$

d. Classify each of the critical numbers as a relative max, relative min, or neither. Find the (x, y) coordinates of the point on the graph at each critical number.

F' changes from + to −

F has a relative max at $x = 0$

$F(0) = 0^3 - 3\cdot 0^2 + 4 = 4$, so the point is $(0, 4)$

F has a relative min at $x = 2$ where F' changes from − to +.

$F(2) = 2^3 - 3\cdot 2^2 + 4 = 0$, so the point is $(2, 0)$

2. We'll do the same sorts of things now with $g(x) = x^4 - (4/3)x^3 - 2x^2 + 4x$.

a. Compute $g'(x)$ and solve to find any critical numbers.
Hint: $(x + 1)$ is one factor of $g'(x)$.

$g'(x) = 4x^3 - 4x^2 - 4x + 4$

factor out the obvious 4, then use long division to factor out the given factor $(x+1)$:

$4(x+1)(x^2-2x+1)$

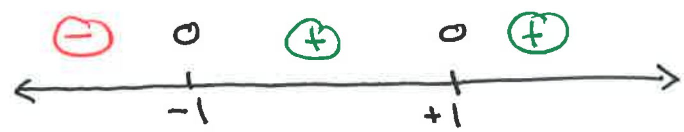
this then factors as $(x-1)^2$

solve: $4x^3 - 4x^2 - 4x + 4 = 0$

(factor) $4(x+1)(x-1)^2 = 0$

crit. numbers $x = -1$ and $x = 1$

b. Determine the sign of $g'(x)$ on the intervals cut out by the critical numbers.



sign of $'(x)$
x

c. Describe the intervals on which $g(x)$ is increasing and decreasing.

g is decreasing on $(-\infty, -1)$

and increasing on $(-1, 1)$ and $(1, \infty)$

d. Classify each of the critical numbers of g, and find the coordinates on the graph of g at each critical number.

g has a relative min. at $x = -1$

g' changes from − to +
g goes like this:

g has neither a min nor a max at $x = +1$, as g' does not change sign there.

3a. (A bit extra - optional!) Return to the cubic function $f(x)$ from problem #1. Solve to find the zeros of $f(x)$. On separate paper, make a large, clear sketch of the graph of f by hand, showing the correct x- and y-intercepts, the correct points at the critical numbers, and the correct increasing/decreasing behavior on all intervals.