Solutions!

Math 118 Calculus Ia **Test 3**

This is a closed-book, closed-notes, no-calculators test. There are 60 points possible.

Fractions and roots in answers are fine; so are negative and fractional exponents.

Use scratch paper as needed, but any work that you want graded should be written legibly on this test paper.

(1 pt) Sign below to indicate your pledge. If your signature is difficult to read, please print your name as well.

I pledge that I will not give, accept, or tolerate others' use of unauthorized aid in completing this work.

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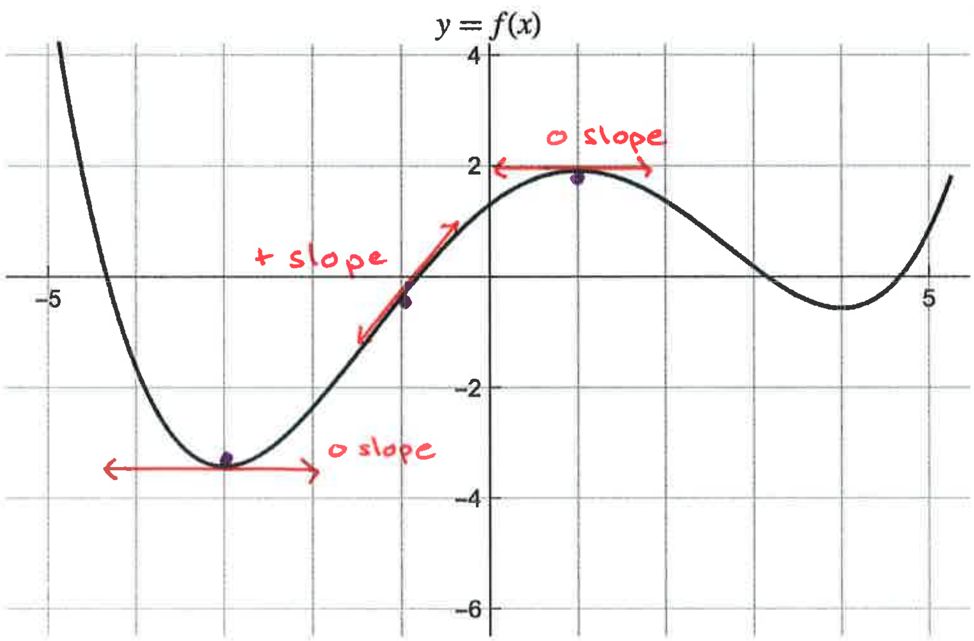
(6 pt) 1. Use the graph of the function f to determine the sign (positive, negative, or zero) of each of the following. Just write +, −, or 0 in the blank beside each part.



− a. $f(-3)$ 0 b. $f'(-3)$

− c. $f(-1)$ + d. $f'(-1)$

+ e. $f(1)$ 0 f. $f'(1)$

(6 pt) 2. Find the derivative of each of the following functions.
Use the power rule; you don't need to set up a limit for these.

a. $f(x) = x^6$ $\quad F'(x) = 6x^5$

either way is fine.

b. $g(x) = \frac{1}{x^8}$ $= x^{-8}$, so $g'(x) = -8x^{-9}$ or $\frac{-8}{x^9}$

c. $h(x) = \sqrt[5]{x}$ $= x^{1/5}$ so $h'(x) = \frac{1}{5}x^{-4/5}$ or $\frac{1}{5x^{4/5}}$

constant multiple

(3 pt) 3. Find the second derivative $g''(x)$, if $g(x) = \frac{1}{20}(x^5 - 10x^3 + 10x^2)$.

$g'(x) = \frac{1}{20}(5x^4 - 30x^2 + 20x)$

$g''(x) = \frac{1}{20}(20x^3 - 60x + 20)$

or simply $x^3 - 3x + 1$

(2 pt) 4. Suppose f is a differentiable function. What's the derivative of $x \cdot f(x)$?
(Just circle the letter of your choice.)

a. $x + f'(x)$ b. $xf(x) + f'(x)$ c. $1 + f'(x)$ d. $f'(x) \cdot f'(f(x))$ e. $xf'(x) + f(x)$

by the product rule.

(2 pt) 5. Suppose f and g are differentiable functions. What's the derivative of $3f(x) + g(x)$?
(Just circle the letter of your choice.)

a. $3f'(x) + g'(x)$ b. $3(f'(x) + g'(x))$ c. $3f(x)g'(x) + 3f'(x)g(x)$

d. $3 + f'(x) + g'(x)$ e. $f'(3)f'(x) + g'(x)$

using the Sum and Constant Multiple rules.

(3 pt) 6a. Complete the definition: The derivative of f is a function $f'(x)$ defined by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(4 pt) b. Apply the definition of the derivative to *set up and evaluate a limit* for the derivative of $f(x) = x^2$.

(**Note:** We know that $f'(x)$ turns out to be $2x$. I want to see the *limit calculation* that proves this.)

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

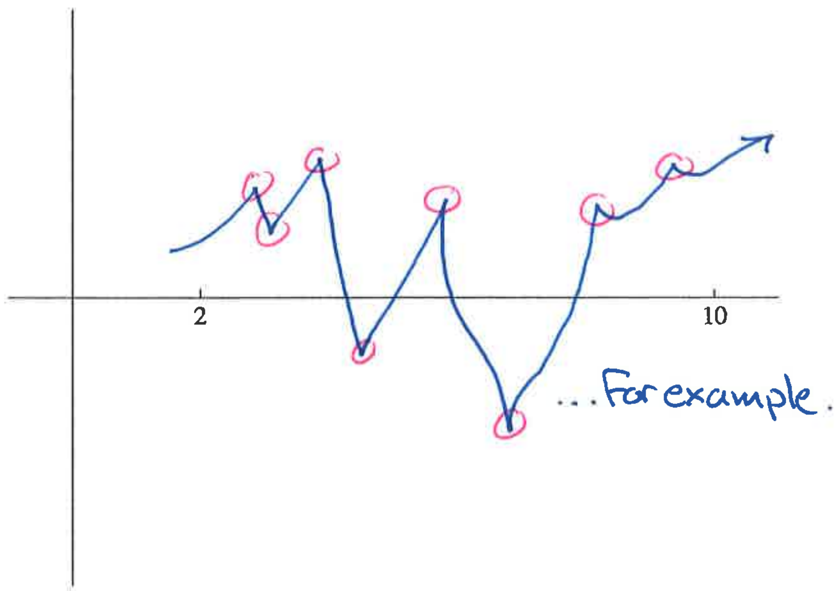
$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\,\Delta x + \Delta x^2 \cancel{-x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(2x + \Delta x)}{\cancel{\Delta x}} = 2x$$

(3 pt) 7. Fill in the blanks with the first two terms (with the highest powers of x) in the expansion of the power:

$$(x + \Delta x)^{25} = \underline{x^{25}} + \underline{25x^{24} \cdot \Delta x} + 300x^{23}\Delta x^2 + 2300x^{22}\Delta x^3 + \text{a lot of other terms}\ldots$$

(3 pt) 8. Use the axes to sketch the graph of a function which is continuous on the interval $[2, 10]$, but has at least one point in the interval at which it is not differentiable.



(4 pt) 9. True/False.

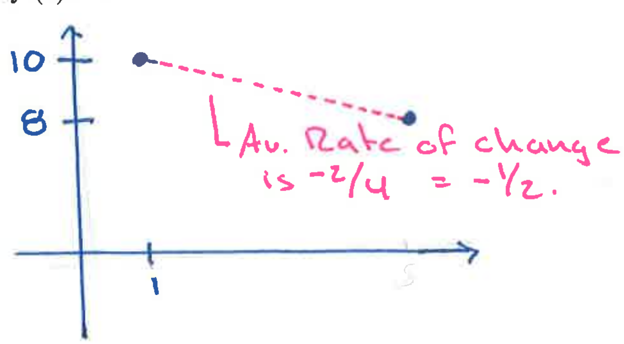
Suppose f is differentiable on the interval $[1, 5]$, and you know that $f(1) = 10$ and $f(5) = 8$.

F a. There must be at least one number c in $(1, 5)$ where $f(c) = -1/2$

T b. There must be at least one number c in $(1, 5)$ where $f'(c) = -1/2$ By MVT

F c. There must be at least one number c in $(1, 5)$ where $f'(c) = 0$

no, not necessarily.



(9 pt) 10. Let $f(x) = x^3 - 3x^2 - 2x + 13$. Find an equation for the tangent line to the graph of f at $x = 1$.

$f(1) = 1^3 - 3\cdot 1^2 - 2\cdot 1 + 13 = 1 - 3 - 2 + 13 = 9$ point: $(1, 9)$.

$f'(x) = 3x^2 - 6x - 2$, so $f'(1) = 3 - 6 - 2 = -5$ slope: -5.

equation of tan. line: $y - 9 = -5(x-1)$

or $y = -5(x-1) + 9$

or $y = -5x + 14$

All fine.

(6 pt) 11. Use the quotient rule to find the derivative of $q(x) = \dfrac{x^2 - 2x - 1}{x + 1}$

Then go on and simplify the derivative by combining like terms in the numerator.

$$q'(x) = \frac{(x+1)(2x-2) - (x^2-2x-1)\cdot 1}{(x+1)^2}$$

quotient rule is done.

(simplifying)

$$= \frac{2x^2 \cancel{+2x} \cancel{-2x} - 2 - x^2 + 2x + 1}{(x+1)^2}$$

$$= \frac{x^2 + 2x - 1}{(x+1)^2}$$

(9 pt) 12. The position function for a braking car is given by

$$f(t) = 12t - 2t^2$$

meters, at time t seconds, up until the time its velocity reaches zero (at which point it stops entirely).

a. Determine the car's velocity in m/s at time $t = 1$ second.

Velocity function: $f'(t) = 12 - 4t$

At $t = 1$: $f'(1) = 12 - 4 \cdot 1 = 8$ m/s.

b. At what time t does the car stop? What is its position at the stopping time? Show your work, but make sure you answer the two questions clearly (including appropriate units with your answers).

It stops when velocity is zero.

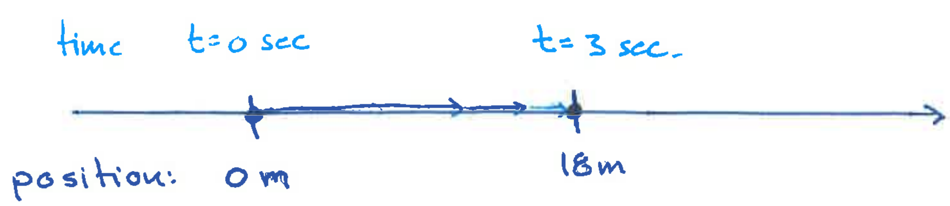
so solve: $12 - 4t = 0$

$4t = 12$

$t = 3$ sec. is the stopping time.

Position when $t = 3$: $f(3) = 12 \cdot 3 - 2 \cdot 3^2$

$= 36 - 18 = 18$ m.



the car moves forward, slowing to a stop at time $t = 3$ sec., 18m ahead of its start position.