Math 118 Calculus I — Test 2

This is a closed-book, closed-notes, no-calculators test. There are 60 points possible. Fractions and square roots in answers are fine, but decimal answers are not needed (or appropriate) for any of the questions. Use scratch paper if you like, but any work that you want graded should be written legibly on this test.

(1 pt) 0. Sign below to indicate your pledge. If your signature is difficult to read, please print your name as well.

I pledge that I will not give, accept, or tolerate others' use of unauthorized aid in completing this work.

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(10 pt) 1. Evaluate the limits (both of which exist; there are no ±∞'s). Make use of the available space to write your solutions clearly.

a. $\lim_{x\to 0} 3x+1+\frac{x+8}{x+4}$ $= 1+\frac{8}{4}$ by direct subst.

$= 1+2$

$= 3.$

b. $\lim_{x\to 3} \frac{x^2-x-6}{x-3}\begin{matrix}\nearrow 0\\ \searrow 0\end{matrix}$ $= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}} = 3+2$ (by direct subst., after cancelling the common factor.)

$= 5$

(4 pt) 2. True/False I

Scoring: 3 correct = 4 points; 2 correct = 2 points; otherwise, 0 points.

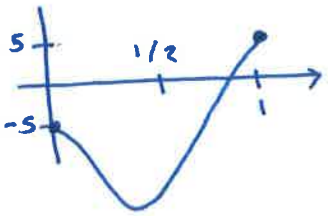
For all three parts of this problem, assume that f is continuous on $[0,1]$, that $f(0)=-5$, and $f(1)=5$.

F a. $f(1/2)$ must be between -5 and 5.

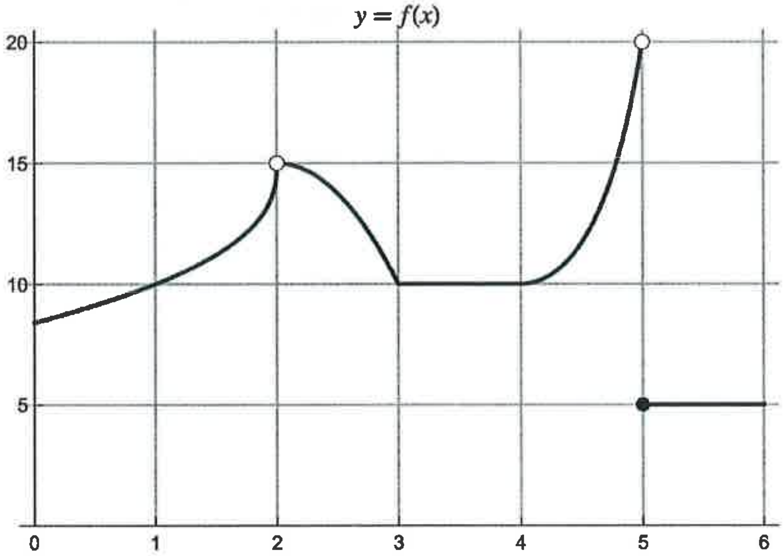
F b. There must be at least one value c in $[0,1]$ where $f(c)=10$.

T c. There must be at least one value c in $[0,1]$ where $f(c)=1$. (Intermediate Value Theorem applies.)

a, b:



For problems #3–4, refer to the graph of the function $f(x)$ shown here.
Note: pay attention to the units on the y-axis.



(6 pt) 3. Graph reading: Determine each of the following values using the graph of $f(x)$ above. If the limit or function value is undefined, or does not exist, just say so.

a. $\lim_{x\to 5^-} f(x) = 20$

b. $f(5) = 5$

c. $\lim_{x\to 5} f(x)$ doesn't exist, since the two 1-sided limits are not equal

d. $\lim_{x\to 2} f(x) = 15$

e. $f(2)$ is undefined

f. $\lim_{x\to 4} \frac{\overset{\nearrow 10}{f(x)}}{\underset{\searrow 18}{x+14}} = \frac{10}{18}$, or $\frac{5}{9}$

(4 pt) 4. True/False II (These questions refer to the function f in the graph above.)
Scoring: 3 correct = 4 points; 2 correct = 2 points; otherwise, 0 points.

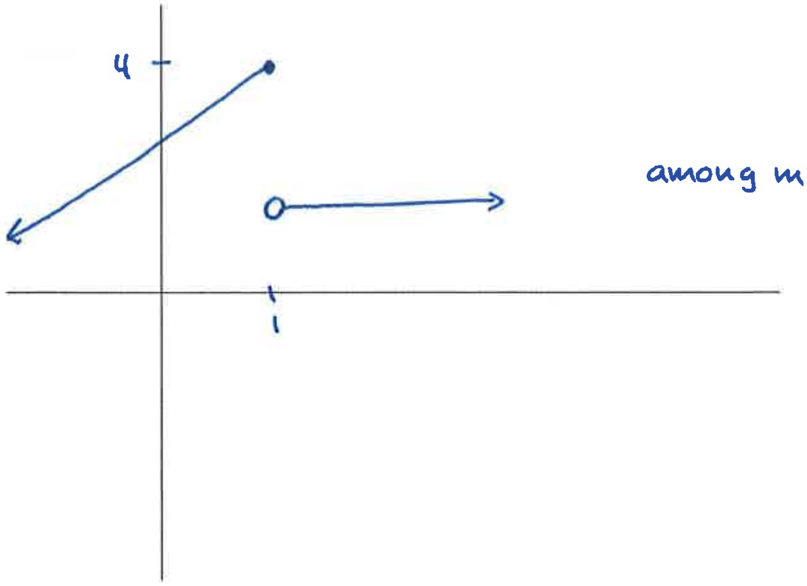
T a. f has a removable discontinuity at 2. removable, since $\lim_{x\to 2} F(x) = 15$

T b. $\lim_{x\to 1} f(x) = f(1)$ f is continuous at $x=1$, visibly.

F c. f is continuous on the open interval $(4, 6)$. there's a (nonremovable) discontinuity in the interval, at $x=5$.

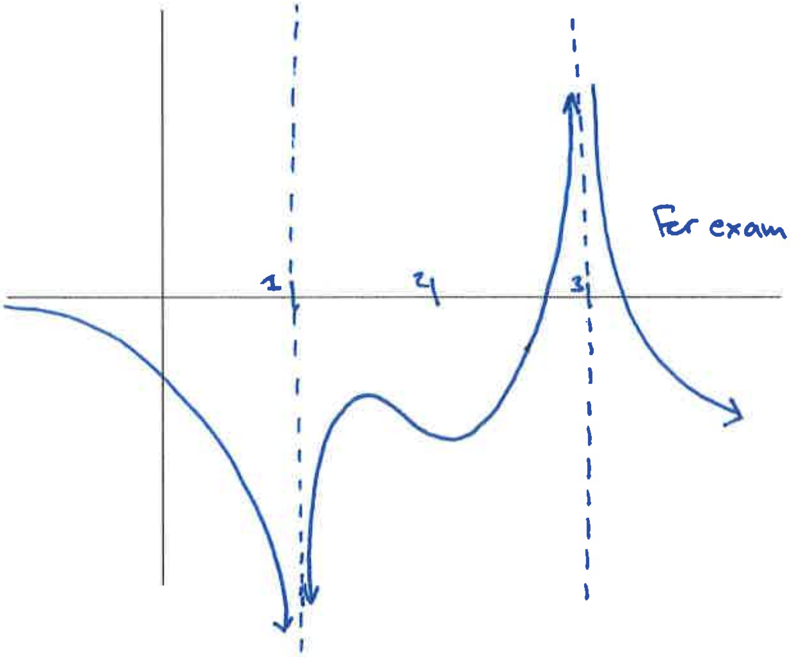
(6 pt) 5. For each part, use the axes to draw the graph of a function satisfying all the given conditions. Make use of the available space and make your drawing as clear as possible. Be sure to mark any necessary values on the x and y axes.

a. $f(1) = 4$ and f has a non-removable discontinuity at 1 (but is continuous everywhere else).



among many possibilities!

b. f is continuous on $(1, 3)$, $\lim_{x\to 1^+} f(x) = -\infty$ and $\lim_{x\to 3^-} f(x) = +\infty$



For example...

(9 pt) 6. Evaluate the following limits. Answer with $+\infty$ or $-\infty$, if appropriate.

a. $\lim_{x\to6+} \frac{x+6 \nearrow 12}{x-6 \searrow 0^+} = +\infty$

b. $\lim_{x\to7} \frac{4 \nearrow 4}{(9+\sqrt{x-7}) \searrow 9} = \frac{4}{9}$

c. $\lim_{x\to10} \frac{1-x^2 \nearrow -99}{(x-10)^2 \searrow 0^+} = -\infty$

(3 pt) 7. Give an example of a function $f(x)$ for which $\lim_{x\to5^-} f(x) = +\infty$ and $\lim_{x\to5^+} f(x) = -\infty$.

Give a simple formula in terms of x. A rational function could be a good example.

For ex., $F(x) = \frac{1}{5-x}$

(5 pt) 8. Evaluate **ONE** of the following limits (both of which exist; there are no ±∞'s)
Cross out the one that you're skipping.

a. $$\lim_{x\to 1} \frac{2x^3 - x - 1}{7x - 7} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

⊛ $$= \lim_{x\to 1} \frac{(2x^2+2x+1)\cancel{(x-1)}}{7\cancel{(x-1)}}$$

$$= \frac{5}{7} \text{ by direct subst.}$$

⊛: Use division to factor:

$$\begin{array}{rl} & 2x^2 + 2x + 1 \\ x-1 & \overline{\big)\, 2x^3 \qquad - x - 1} \\ & -(2x^3 - 2x^2) \\ \hline & \quad 2x^2 - x - 1 \\ & \quad -(2x^2 - 2x) \\ \hline & \qquad +x - 1 \\ & \qquad -(x - 1) \\ \hline & \qquad\quad 0. \end{array}$$

b. $$\lim_{x\to 0} \frac{\sqrt{13+x} - \sqrt{13-x}}{2x} \cdot \frac{\sqrt{13+x} + \sqrt{13-x}}{\sqrt{13+x} + \sqrt{13-x}}$$

$$= \lim_{x\to 0} \frac{(13+x)-(13-x)}{2x\,(\text{ stuff}).}$$

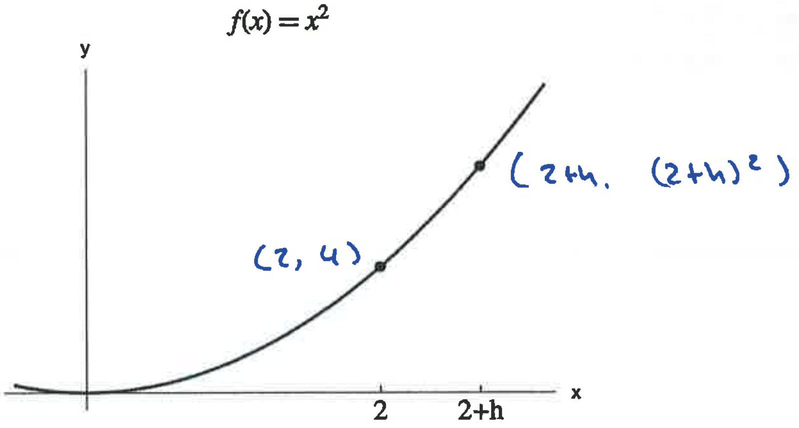
$$= \lim_{x\to 0} \frac{\cancel{2x} \quad 1}{\cancel{2x}\left(\sqrt{13+x} + \sqrt{13-x}\right)}$$

$$= \frac{1}{2\sqrt{13}} \text{ by direct subst.}$$

(4 pt) 9. Expand the following powers and simplify the expansion by combining like terms.

a. $(x+h)^2$ $= (x+h)(x+h) = x^2 + 2xh + h^2$

b. $(x+h)^3$ $= (x^2 + 2xh + h^2)(x+h) = x^3 + 3x^2h + 3xh^2 + h^3$



(9 pt) 10. Referring to the graph shown above:

a. Express the slope of the line connecting the two marked points in terms of h.

b. Evaluate the limit of that slope (your answer from part a) as $h \to 0$.

slope for part a

$$\lim_{h\to 0} \boxed{\frac{(2+h)^2 - 4}{h}}$$

$$= \lim_{h\to 0} \frac{\cancel{4+} 4h + h^2 \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = 4$$ by direct subst.