Solutions.

Math 118 Calculus I **Test 1**

This is a closed-book, closed-notes, no-calculators test. There are 60 points possible. Fractions and square roots in answers are fine, but decimal answers are not needed (or appropriate) for any of the questions. Use scratch paper if you like, but any work that you want graded should be written legibly on this test.

(1 pt) When you are finished, sign below to indicate your pledge. If your signature is difficult to read, please print your name as well.

I pledge that I have not given, received, or tolerated others use of unauthorized aid in completing this work.

(6 pt) 1a. Find the slope of the line passing through $(6, -3)$ and $(19, 2)$.

$$\frac{2-(-3)}{19-6} = \frac{5}{13}$$

b. Write an equation for a line with slope $\frac{1}{2}$, passing through the point $(5, -12)$.

$$y + 12 = \frac{1}{2}(x-5)$$

or $y = \frac{1}{2}(x-5) - 12$ or $\frac{1}{2}x - 14\frac{1}{2}$

(6 pt) 2. Let f be the function defined by $f(x) = 2x^2 - x$.

a. Compute the net change in f over the interval $[2, 4]$.

$$f(4) - f(2) = (2\cdot 4^2 - 4) - (2\cdot 2^2 - 2)$$
$$= 28 - 6$$
$$= 22$$

b. Compute the average rate of change in f over the interval $[2, 4]$.

$$\frac{f(4) - f(2)}{4-2} = \frac{22}{2} = 11$$

(9 pt) 3. Solve to find all values of x that satisfy the equation.
Give exact answers, not decimal approximations.

a. $2(x+5)-7=3(x-2)$

$2x+3 = 3x-6$

$9 = x$

b. $x^{10}-2x^9=0$ $\quad x^9(x-2)=0$

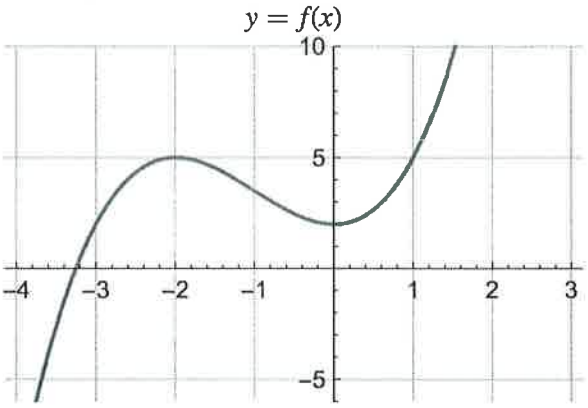
either $x=0$ or $x=2$.

c. $8x=5+2x^2$ $\quad 2x^2-8x+5=0$

Quadratic Formula: $x = \dfrac{8 \pm \sqrt{64-4\cdot 5\cdot 2}}{2\cdot 2}$

$= \dfrac{8\pm\sqrt{24}}{4}$ - fine

or simplified a bit, $x = \dfrac{4\pm\sqrt{6}}{2}$



(12 pt) 4. Consider the graph of the function f shown above. **Assume that $f(x)$ is a polynomial of degree three.** Answer the following questions:

a. How many real zeros does f have? 1

b. How many turning points does f have? 2

c. What is the net change in f over the interval $[-2, 0]$? $2 - 5 = -3$
$f(0) - f(-2)$
the order of subtraction matters.

d. What is the average rate of change in f over the interval $[-2, 0]$? $\frac{2-5}{0-(-2)} = \frac{-3}{2}$

e. Another function g is defined by $g(x) = f(x) - 3$. How many real zeros does g have? 3
The graph of $g(x)$ is like the graph of $f(x)$ shifted down 3 units.

f. Is it possible to determine the sign (positive or negative) of the leading coefficient of $f(x)$?
Explain briefly: yes, it must be positive for f to rise to the right.

(4 pt) 5. True/False I (These questions refer to the function f in the graph above)
Scoring: 3 correct = 4pt, 2 correct = 2pt; otherwise, 0pt.

T a. f is decreasing on the interval $[-2, 0]$

F b. f is increasing on the interval $[-1, 1]$ – the net change over $[-1,1]$ is positive, but f is not increasing over the whole interval.

F c. f has the same average rate of change over every interval.
2 if this were true, the graph would be a line, with constant slope.

(9 pt) 6. Function h is defined by $h(x) = 5x - 2x^2$.

a. Evaluate $h(x+4)$. Simplify the result by expanding a power and then combining like terms.

think: $h(\square) = 5(\square) - 2(\square)^2$

so $h(x+4) = 5(x+4) - 2(x+4)^2$

$= 5x + 20 - 2(x^2 + 8x + 16)$

$= -2x^2 - 11x - 12$

b. Write an equation for the secant line which meets the graph of h at $x = -1$ and $x = 2$.

$h(-1) = -7$, so points: $(-1, -7)$ and $(2, 2)$ $\rightarrow$ slope: $\frac{2-(-7)}{2-(-1)} = 3$

i $h(2) = 2$

line: $y - 2 = 3(x-2)$

or $y + 7 = 3(x+1)$

or $y = 3x - 4$ among other ways of writing it.

(6 pt) 7. Give an example of a function with the given properties.
(Give a simple formula in terms of x, such as $x^2 + 1$ or $(2x+3)^4$ or whatever.)

a. A function which has the same average rate of change over every interval.

For ex., $f(x) = 7x$ any linear function, including constant functions like $f(x) = 5$

b. A polynomial which rises to the left, falls to the right, and has $x = 2$ as a zero.

For ex., $f(x) = -x + 2$ or $-x^3 + 8$ odd degree, negative leading coeff. and $x = 2$ is a zero.

c. A function which is decreasing over every interval.

For ex., $f(x) = 10 - x$ - any linear function with negative slope is good. There are other possibilities, too.

(4 pt) 8. Simplify (by canceling a common factor).

! (a.) $\dfrac{t^2-4t}{t^6+8t}$ $= \dfrac{t(t-4)}{t(t^5+8)} = \dfrac{t-4}{t^5+8}$

(b.) $\dfrac{x^2h-x^2}{h-1}$ $= \dfrac{x^2\cdot(h-1)}{(h-1)}$ = simply x^2

(4 pt) 9. True/False II

Scoring: 3 correct = 4pt, 2 correct = 2pt; otherwise, 0pt.

F a. Every polynomial has at least one real zero. For ex., x^2+1 has no zeros.

F b. Every polynomial of odd degree has at least one turning point. for ex., x^3 has none.

F (c.) A polynomial of even degree must have an even number of zeros.
For ex., x^2 has exactly 1 zero