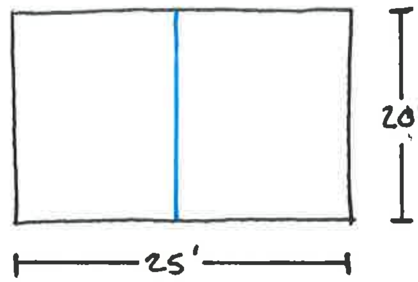
Prob 7 Solutions!

1. The optimal (minimum-cost) pen should be constructed as follows:



The cost will be \$200. (\$180 of the expensive fencing around the outside, plus \$20 of the cheap fencing for the interior.)

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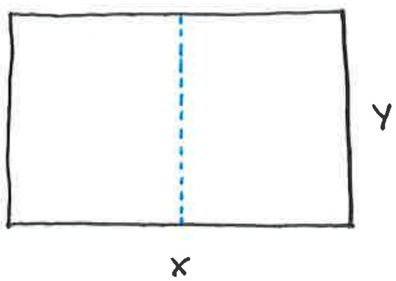
sol'n



① cost $C = 4x + 5y$ (dollars)

② $xy = 500$ sq. ft

so $y = \frac{500}{x}$.

Rewrite: $C(x) = 4x + \frac{2500}{x}$ expresses cost as a function of x alone.

diff., and find the critical value of x:

$$C'(x) = 4 - \frac{2500}{x^2}$$

solve: $4 - \frac{2500}{x^2} = 0$

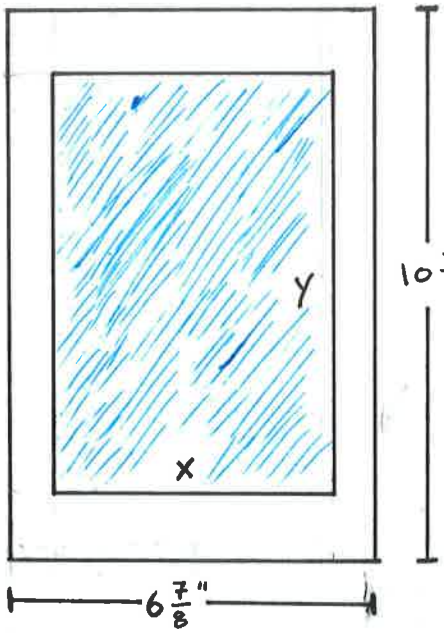
$x^2 = \frac{2500}{4}$ so $x = 25$ ft.

So the corresponding value of y is $y = 500/25 = 20$ ft.

and the cost is $C = 4 \cdot 25 + 5 \cdot 20 = \$200$.

Prob 7 solutions!

2.



The optimal (minimum-area) page should be 6.9" wide (left-to-right) and 10.35" tall (top-to-bottom). (That's about $6\frac{7}{8} \times 10\frac{3}{8}$, which are probably the nearest fractions you could measure with a ruler.)

The printed region will be $x = 4\frac{7}{8}$" by $y = 7\frac{3}{8}$"

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soln Let x and y be the width & height of the printed part, as shown above.

Then ① Area of the page, $A = (x+2)(y+3)$ (to be minimized)

② $xy = 36$ sq. in.

so $y = \frac{36}{x}$,

and ③ $A(x) = (x+2)\left(\frac{36}{x} + 3\right)$

or $A(x) = 3x + \frac{72}{x} + 42$ expresses A as a function of x alone.

Then $A'(x) = 3 - \frac{72}{x^2}$

solve: $3 - \frac{72}{x^2} = 0$ so $x = \sqrt{24} = 2\sqrt{6}$ is the critical, minimizing value for x.

the corresponding value for y is

$y = 36/2\sqrt{6} = 3\sqrt{6}$

add the margins

So the exact dimensions of the page are $2 + 2\sqrt{6}$ in $\times$ $3 + 3\sqrt{6}$ in

or about 6.90 in $\times$ 10.35 in.

interesting to note that the optimal y value is exactly $\frac{3}{2}$ times the x value. It's easier to see that relation in the exact values $2\sqrt{6}$ and $3\sqrt{6}$ than in the decimal versions. So both forms are useful.