Prob. 5

1a. $f(1) = 1^3 - 1 - 1 = -1$
and $f(4) = 4^3 - 4 - 1 = 59$ $\Big\}$ so the two points are $(1,-1)$ and $(4,59)$.

The slope is $\frac{59-(-1)}{4-1} = \frac{60}{3} = 20$.

point

So the secant line is $y - (\underline{-1}) = \underline{20}(x - \underline{1})$

slope.

or $y = 20(x-1)-1$,
if you isolate y.

b. $f'(x) = 3x^2 - 1$ (using derivative rules)
so, solve: $3x^2 - 1 = 20$
$3x^2 = 21$,
$x^2 = 7$ so $x = -\sqrt{7}$ or $\sqrt{7}$,
but only $\sqrt{7}$ is between 1 and 4.

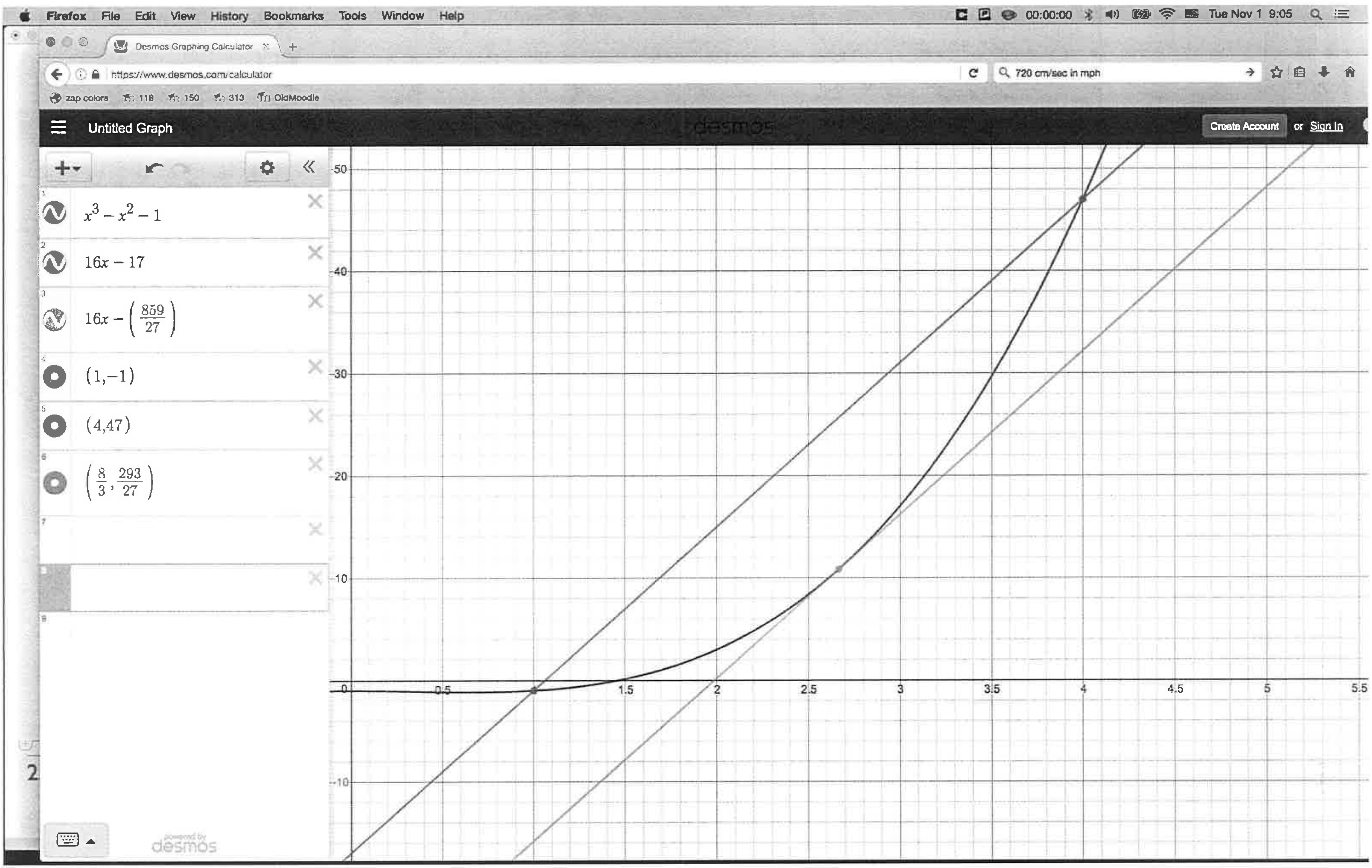
At $x = \sqrt{7}$, $y = f(\sqrt{7}) = \sqrt{7}^3 - \sqrt{7} - 1 = 6\sqrt{7} - 1$

point: $(\sqrt{7}, 6\sqrt{7} - 1)$

And the slope of the tangent line is
$f'(\sqrt{7}) = 3 \cdot \sqrt{7}^2 - 1 = 21 - 1 = 20$ as we expected

so the tangent line is

$$y = 20(x - \sqrt{7}) + 6\sqrt{7} - 1.$$



Firefox File Edit View History Bookmarks Tools Window Help
00:00:00 Tue Nov 1 9:05
Desmos Graphing Calculator
https://www.desmos.com/calculator
720 cm/sec in mph
zap colors 118 150 313 OldMoodle
Untitled Graph
desmos
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$x^3 - x^2 - 1$
$16x - 17$
$16x - \left(\frac{859}{27}\right)$
$(1,-1)$
$(4,47)$
$\left(\frac{8}{3}, \frac{293}{27}\right)$
50
40
30
20
10
-10
0
0.5
1.5
2
2.5
3
3.5
4
4.5
5
5.5
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Prob 5

2a. Solve: $\frac{16}{3}(27t^2 - t^3) = \frac{8}{5}(105t^2 - 4t^3)$

mult. by 15 to clear the fractions:

$$80(27t^2 - t^3) = 24(105t^2 - 4t^3)$$
$$10(27t^2 - t^3) = 3(105t^2 - 4t^3)$$

gather terms on one side:

$$45t^2 - 2t^3 = 0$$

$$t^2(45 - 2t) = 0$$

so either $t=0$ or $t = 45/2$ ($= 22.5$) seconds.

initial time when they're both on the ground.

So they have the same height at time $t = \frac{45}{2}$ sec., and the height at that time is

$$P_1(45/2) = 12{,}150 \text{ cm.} \quad \text{(just under 400 ft.)}$$

calculator!

you can check your result by computing $P_2(45/2)$, too. they should be the same.

2b. The velocity functions are

$$p_1'(t) = \frac{16}{3}(54t - 3t^2) \quad \text{and} \quad p_2'(t) = \frac{8}{5}(210t - 12t^2)$$

so, solve: $\frac{16}{3}(54t - 3t^2) = \frac{8}{5}(210t - 12t^2)$

clear fractions as in part (a): $10(54t - 3t^2) = 3(210t - 12t^2)$

gather, & factor... $90t - 6t^2 = 0$

$t(90 - 6t) = 0,$

so either

$\cancel{t=0}$ or $t = 90/6 = \underline{15 \text{ sec.}}$

doesn't fit the problem.

the time we're looking for.

At time $t = 15$ sec after launch, both rockets have a velocity of

$p_1'(15) = (\ldots \text{calculator} \ldots) = 720$ cm/sec.

The height of the first rocket at that time is

$p_1(15) = \cdots = 14{,}400$ cm.

calculator!

And the second rocket is at height

$p_2(15) = \cdots = 16{,}200$ cm.