118 Prob 4 Solutions

1a. $f(x) = 4x^2 - 6x$ think: $f(\boxed{}) = 4(\boxed{})^2 - 6(\boxed{})$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4(x+\Delta x)^2 - 6(x+\Delta x) - (4x^2 - 6x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{4x^2} + 8x\cdot\Delta x + 4\Delta x^2 \cancel{-6x} - 6\Delta x \cancel{-4x^2} \cancel{+6x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\cdot(8x + \overbrace{4\Delta x}^{\to 0} - 6)}{\cancel{\Delta x}}$$

$$= 8x - 6.$$ In brief, $f'(x) = 8x - 6.$

1b. $f(x) = \sqrt{x+7}$ think: $f(\boxed{}) = \sqrt{\boxed{} + 7}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\overbrace{\sqrt{x+\Delta x+7}}^{f(x+\Delta x)} - \overbrace{\sqrt{x+7}}^{f(x)}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+7} + \sqrt{x+7}}{\sqrt{x+\Delta x+7} + \sqrt{x+7}}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x} + \cancel{\Delta x} \cancel{+7} \cancel{-(x+7)}}{\cancel{\Delta x}(\sqrt{x+\Delta x+7} + \sqrt{x+7})}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\underbrace{\Delta x}_{\to 0}+7} + \sqrt{x+7}}$$

$$= \frac{1}{2\sqrt{x+7}}.$$

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2. $f(x) = \sqrt{3x^2+1}$ $f'(x) = \dfrac{3x}{\sqrt{3x^2+1}}$

at $x=4$, $y = f(4) = \sqrt{3\cdot 4^2+1} = \sqrt{49} = 7$,
so the line must go through $\underline{(4,7)}$.
point

the slope of the tangent line at $x=4$ is

$$f'(4) = \frac{3\cdot 4}{\sqrt{49}} = \underline{\frac{12}{7}}$$
slope

Thus the tangent line is $y-7 = \frac{12}{7}(x-4)$

or $y = \frac{12}{7}(x-4)+7$,
if you like.