116 Prob 3 soln's.

1a. $\lim_{x\to\sqrt{7}} \frac{x^2-7}{x^4-7x^2}$ ($\to 0$ / $\to 0$)

better factor!

$= \lim_{x\to\sqrt{7}} \frac{(x^2-7)}{x^2(x^2-7)}$

$= \lim_{x\to\sqrt{7}} \frac{1}{x^2} = \frac{1}{7}$ by direct substitution.

1b. $\lim_{x\to 5} \frac{(\sqrt{x+4}-3)}{(x-5)} \cdot \frac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)}$ ($\to 0$ / $\to 0$)

$= \lim_{x\to 5} \frac{(x+4)-9}{(x-5)(\sqrt{x+4}+3)}$

$= \lim_{x\to 5} \frac{(x-5)\ 1}{(x-5)(\sqrt{x+4}+3)}$

$= \frac{1}{\sqrt{9}+3}$ by d.s., or simply $\frac{1}{3+3} = \frac{1}{6}$.

2a. $\lim_{\Delta x\to 0} \frac{((x+\Delta x)^3 - x^3)}{(\Delta x)}$ ($\to 0$ / $\to 0$) $= \lim_{\Delta x\to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - x^3}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\Delta x\cdot(3x^2 + 3x\Delta x + \Delta x^2)}{\Delta x}$

$= \lim_{\Delta x\to 0} 3x^2 + \underbrace{3x\Delta x}_{\to 0} + \underbrace{\Delta x^2}_{\to 0}$

$= 3x^2$.

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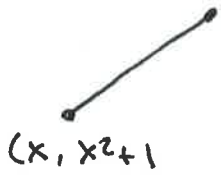
3a. The full coordinates of the points are

$(x+\Delta x,\ (x+\Delta x)^2+1)$.



so the slope of the line is $\dfrac{(x+\Delta x)^2+1 - (x^2+1)}{x+\Delta x - x}$

b. $\lim_{\Delta x \to 0} \dfrac{(x+\Delta x)^2+1-(x^2+1)}{x+\Delta x - x}$ ($\to 0$ / $\to 0$)

$= \lim_{\Delta x \to 0} \dfrac{x^2+2x\Delta x+\Delta x^2+1-x^2-1}{x+\Delta x - x}$

$= \lim_{\Delta x \to 0} \dfrac{2x\cdot\Delta x+\Delta x^2}{\Delta x}$

↙ Factor out Δx, & cancel the common factor

$= \lim_{\Delta x \to 0} 2x+\Delta x$

$= 2x.$