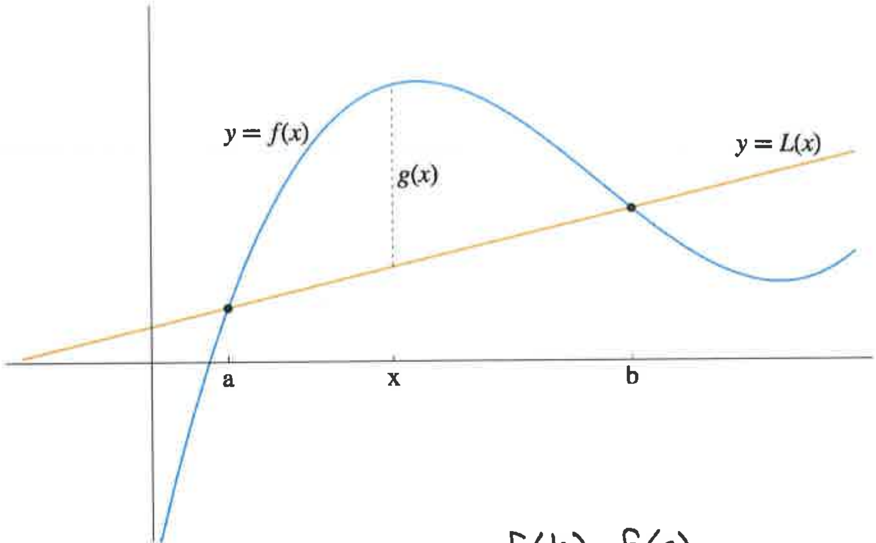
Assume that $f(x)$ is a differentiable function on the interval $[a, b]$, as shown in the graph.



The slope of the secant line, expressed in terms of f, is $\frac{f(b)-f(a)}{b-a}$

If we call that slope m for short, then the linear function $L(x)$ describing the secant line is given by

$$L(x) = m\cdot(x-a) + f(a).$$

$m\cdot(x-b) + f(b)$ would work, too.

Let $g(x)$ denote the difference between the functions $f(x)$ and $L(x)$. In symbols, that is,

$$g(x) = f(x) - L(x)$$

By the Sum/Difference rule for derivatives, we know that g is differentiable on $[a, b]$ and

$$(*) \quad g'(x) = f'(x) - L'(x)$$

And since $L(x)$ is a linear function with slope m, that can be written more simply as

$$g'(x) = f'(x) - m$$

Also, we can see that $g(a) =$ 0 and $g(b) =$ 0. – since there's no difference in height at $x=a$ and at $x=b$.

So, Rolle's Theorem applies to the function g on the interval $[a,b]$.

That lets us conclude that there is at least one point c in (a,b) where $g'(c)=0$.

But $g'(c) = f'(c) - m$. So we can conclude that $f'(c) = m$, qed.

look back to (*).