118 - "Points & Interest"
Answers / solutions.

- These refer to the function $f(x) = x^3 - 6x$; you can see the graph on the original handout.

1. On the graph, ~~the~~ $y = f(x)$, so these are asking about the y-coordinate of a point on the graph:

   a. $f(-3)$ is negative
   b. $f(-1)$ is pos.
   c. $f(0)$ is 0
   d. $f(1)$ is neg.
   e. $f(3)$ is pos.

2. But $f'(x)$ is the slope of the tangent line to the graph (at the given x-value):

   a. $f'(-3)$ is $+$
   b. $f'(-1)$ is $-$
   c. $f'(0)$ is $-$
   d. $f'(1)$ is $-$
   e. $f'(3)$ is $+$

   sketch the tangent lines in at those various x values!

3. At the two marked points, the tangent line is horizontal, so the slope at those points is 0.

4. To get the derivative:

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - 6(x+\Delta x) - [x^3 - 6x]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 \cancel{-6x} - 6\Delta x \cancel{-x^3} \cancel{+6x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(3x^2 + \overbrace{3x\Delta x}^{\to 0} + \overbrace{\Delta x^2}^{\to 0} - 6)}{\cancel{\Delta x}} = 3x^2 - 6.$$

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5. Using that formula for $f'(x)$,

"The slope of the tangent line at x is zero"

becomes the equation

$$3x^2 - 6 = 0,$$

which we can solve:

$$3x^2 = 6$$

so $x^2 = 2$

and $x = \sqrt{2}$ or $x = -\sqrt{2}$.

6. The y-coord. of points on the graph is given by $y = x^3 - 6x$, so

oops!

at $x = \sqrt{2}$, $y = (\sqrt{2})^3 - 6\sqrt{2} = 2\cancel{4}\sqrt{2} - 6\sqrt{2} = -4\sqrt{2}$.

and at $x = -\sqrt{2}$, $y = (-\sqrt{2})^3 - 6(-\sqrt{2}) = -2\sqrt{2} + 6\sqrt{2} = 4\sqrt{2}$.

So our two points are $(\sqrt{2}, -4\sqrt{2})$ and $(-\sqrt{2}, 4\sqrt{2})$.