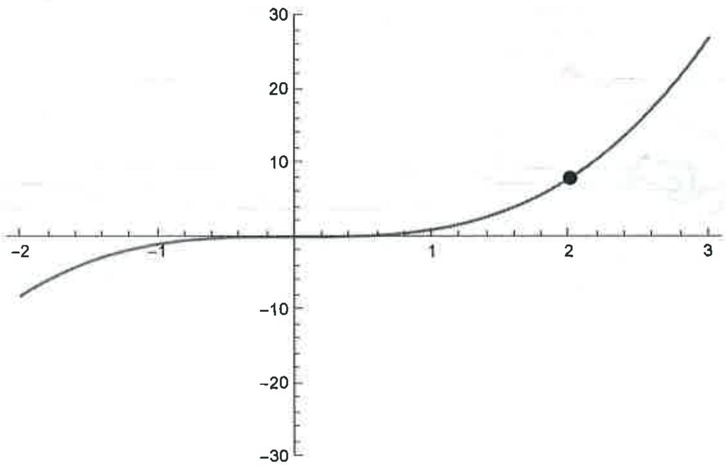
Functions and their Derivatives (I)

Example 1. $f(x) = x^3$



1. Give the formula we computed for the derivative of this function: $f'(x) =$ $3x^2$

2. Give the coordinates of the point marked on the graph (the x-coordinate is 2): $(2, 8)$

$f(2) = 2^3 = 8$ gives the y-coordinate.

3. What's the *slope* of the tangent line at the marked point? (Use the work we've already done!)

The slope of the tan. line at $x=2$ is $f'(2) = 3 \cdot 2^2 = 12$.

4. Give an *equation* for the tangent line to the graph at x=2.

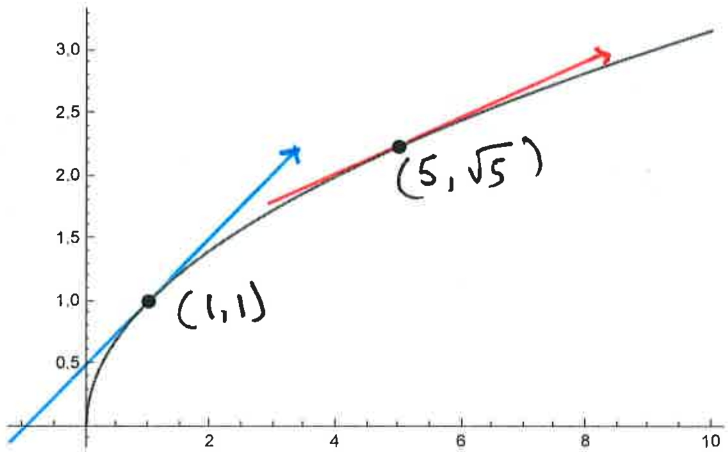
point $(2, 8)$ slope 12

point-slope eq'n: $y - 8 = 12(x-2)$

or $y = 12(x-2) + 8$.

Functions and their Derivatives (2)

Example 2. $f(x) = \sqrt{x}$



1. Give a formula for the derivative of this function (from the work we did in class).

$$F'(x) = \frac{1}{2\sqrt{x}}$$

2. Compute the slope of the secant line connecting the points on the graph at x=1 and x=5.
   (Simplify your answer, but leave it in terms of square roots - no decimal approximations.)

$$\frac{\sqrt{5} - 1}{4}$$

3. Make a visual estimate (without any calculation): Which *tangent* line has the larger slope, the tangent line at x=1 or the tangent line at x=5?

Sketching in the tangent lines, the blue tangent line at $x=1$ has a larger slope than the red tangent line at $x=5$.

3a. Compute the exact slope of the *tangent* line to the graph at x=1.
   Note: You don't need to evaluate a limit for this problem. Use the work we've already done!

$$F'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

b. And compute the exact slope of the tangent line to the graph at x=5.

$$F'(5) = \frac{1}{2\sqrt{5}}$$

c. Write equations for both tangent lines (one for the tangent at x=1 and one for the tangent at x=5).

at $x=1$: $y = \frac{1}{2}(x-1) + 1$ at $x=5$: $y = \frac{1}{2\sqrt{5}}(x-5) + \sqrt{5}$