You'll probably need some extra paper to complete some of these!

1. Determine the *sign* (positive or negative) of each of the following quantities, based on the given information about x.

$\ominus$ a. $(x-3)$ if x is just slightly less than 3.

$\oplus$ b. $(x-5)^3$ if x is just slightly greater than 5.

$\ominus$ c. $(x+1)(x-1)$ if x is just slightly less than 1.
$+ \cdot -$

$\oplus$ d. $\dfrac{x-10}{x-19}\,\dfrac{-}{-}$ if x is just slightly less than 10.

$\ominus$ e. $\dfrac{-2}{(x-30)^4}\,\dfrac{-}{+}$ if x is very close (but not equal) to 30.

$\oplus$ f. $\dfrac{\overset{+\ \cdot\ -}{(x+4)(x-7)}}{\underset{+\ \cdot\ -}{x^2(x-11)}}$ if x is very close (but not equal) to 0. in all, it is $\frac{-}{-}$, hence $+$.

2. Evaluate the following limits. Use $+\infty$ or $-\infty$ if appropriate.

a. $\lim\limits_{x\to 8} \dfrac{12}{(x-8)^2}$ $\quad \to 12,\ \to 0^+$ $\quad = +\infty$

b. $\lim\limits_{x\to 1^-} \dfrac{x^2+2x-3}{x^2+x+1}$ $\quad \to 0,\ \to 3$ $\quad = \frac{0}{3} = 0.$ direct substitution works fine, the function is continuous at $x=1$.

c. $\lim\limits_{x\to 25} \dfrac{(x-13)}{(x-25)^2}$ $\quad \to 12,\ \to 0^+$ $\quad = +\infty.$

d. $\lim\limits_{x\to 3^+} \dfrac{x^2-3}{(x-3)(x+3)}$ $\quad \to 6,\ \to 0^+$ $\quad = +\infty.$

* I factored by dividing:

$$\begin{array}{r|l} & x^2 \quad +2x \quad +4 \\ \hline x-2 & x^3 \qquad\qquad\quad -8 \\ & -(x^3-2x^2) \\ \hline & = \quad 2x^2 \qquad -8 \\ & -(2x^2-4x) \\ \hline & = \quad 4x-8 \\ & -(4x-8) \\ \hline & \qquad 0. \end{array}$$

3. For the function $f(x) = \dfrac{x^3-8}{x^2-4}$,

a. Identify all points at which $f(x)$ is undefined.

b. Identify all vertical asymptotes of the function.
Compute one-sided limits at each vertical asymptote.

c. Identify all points at which the function has a removable discontinuity.

d. Identify the intervals on which the function is continuous.

a. $f(x)$ is undefined where $x^2-4=0$
so $x=2$ and $x=-2$.

(b + c). $\displaystyle \lim_{x\to 2} \frac{x^3-8 \to 0}{x^2-4 \to 0} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)^* \to 12}{\cancel{(x-2)}(x+2) \to 4} = \frac{12}{4} = 3.$

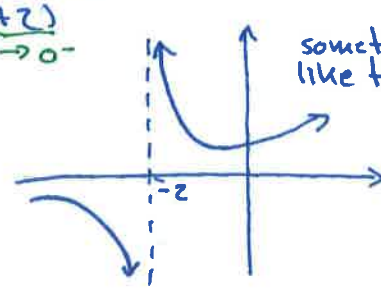
since this limit exists,
f has a <u>removable</u> discontinuity at 2

$\displaystyle \lim_{x\to -2} \frac{x^3-8 \to -16}{x^2-4 \to 0}$ ... so there is a vertical asymptote at 2.

1-sided limits:

$$\lim_{x\to -2^+} \frac{x^3-8}{x^2-4} = \lim_{x\to -2^+} \frac{x^3-8 \to -16}{\underbrace{(x-2)}_{\to -4}\underbrace{(x+2)}_{\to 0^+}} = +\infty.$$

$$\lim_{x\to -2^-} \frac{x^3-8}{x^2-4} = \lim_{x\to -2^-} \frac{(x^3-8) \to -16}{\underbrace{(x-2)}_{\to -4}\underbrace{(x+2)}_{\to 0^-}} = -\infty.$$



something like this.

d. f is continuous on $(-\infty, -2)$,
$(-2, 2)$,
and $(2, +\infty)$.

4. Repeat parts (a)-(d) of the above problem with the function $f(x) = \dfrac{5x^4 - 16x^3}{8x^4 - x^3}$

a. undef'd (hence discontinuous) where

$$8x^4 - x^3 = 0$$

Factor: $x^3(8x-1) = 0$

$x = 0$ or $x = 1/8$ are the only two discontinuities.

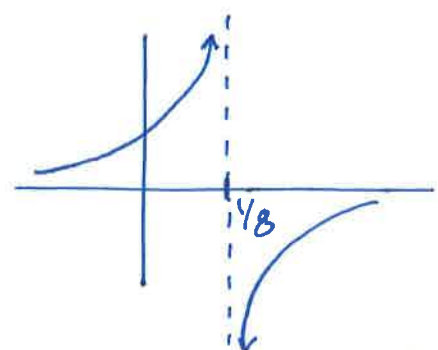
(b+c) At $x=0$: $\displaystyle\lim_{x\to 0} \frac{5x^4 - 16x^3 \nearrow 0}{8x^4 - x^3 \searrow 0}$ Factor!

$$= \lim_{x\to 0} \frac{\cancel{x^3}(5x-16) \nearrow -16}{\cancel{x^3}(8x-1) \searrow -1} = 16.$$

so there's a <u>removable discontinuity</u> at $x=0$.

At $x = 1/8$, the 1-sided limits are:

$$\lim_{x\to 1/8^+} \frac{\cancel{x^3}(5x-16) \nearrow \ominus}{\cancel{x^3}(8x-1) \searrow 0^+} = -\infty.$$



$$\lim_{x\to 1/8^-} \frac{\cancel{x^3}(5x-16) \nearrow \ominus}{\cancel{x^3}(8x-1) \searrow 0^-} = +\infty.$$

something like this near $x = 1/8$.

(we have a vertical asymptote at $x = 1/8$.)